Teacher Edition

Mathematics

Indiana's Academic Standards

Adopted by the
Indiana State
Board of Education
May 2009

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The academic achievement and career preparation of all Indiana students will be the best in the United States and on par with the most competitive countries in the world.

That, simply stated, is the vision for education in Indiana, and it begins with world-class standards for what students should know and be able to do.

Indiana's World-Class Standards

Developed with the assistance of K-12 teachers, community members and content experts at the university level, Indiana's Academic Standards for Mathematics have been ranked among the very best in the nation by such respected authorities as Achieve, Inc. and the Thomas B. Fordham Foundation. Widespread recognition for establishing clear, rigorous expectations at each grade level is significant, but the more critical challenge remains: ensuring these standards result in student learning.

With that goal in mind, this edition of Indiana's Academic Standards for Mathematics integrates two important elements: **core standards** and **process standards**. The core standards highlight the "big ideas" at each grade level to help prioritize instruction around the essential concepts students must learn to advance. The process standards offer a comprehensive approach to classroom instruction. Examples are also provided to illustrate concepts being addressed by the standards. Together, the core standards encompass what students must learn while the process standards address how the content should be taught.

Math Matters

There is no question that math opens the door to greater opportunities. A solid grounding in math is linked to expanded career options and increased earning power. Consider, for instance, that students who complete Algebra II are more than twice as likely to graduate from college, and they comprise three-fourths of adults in the top-paying quarter of jobs.

Math is a passport to prosperity, and it also develops the critical thinking and analytic skills needed to tackle everyday problems (e.g., managing personal finances, selecting a health care plan, choosing a mortgage, planning for retirement). Clearly, rigorous math preparation can no longer be considered the sole domain of future engineers and scientists.

Expect More

Meeting higher expectations leads to greater rewards and opportunities for Indiana students. By setting specific goals, everyone wins. Teachers have clear targets, students know what's expected, and parents have detailed information about their children's strengths and weaknesses. The standards are a good way to engage parents in meaningful dialogue about their role in supporting student progress. It is also important to talk to students about those expectations to help them take responsibility for their learning.

Indiana cannot afford to let students give up on math because it does not come easily nor perpetuate the belief shared by many adults that some simply cannot be "good" at math. Hoosier students' peers in other countries generally accept the opposite to be true. Math can be challenging, but it will be learned given the necessary motivation and determination.

Learn More

To learn more about Indiana's Academic Standards for Mathematics and related resources designed to help students reach these essential expectations, visit www.indianamath.gov and www.doe.in.gov/standards. To learn more about the Indiana Department of Education's action plan for raising student achievement, visit www.doe.in.gov/actionplan.

Source: National Mathematics Advisory Panel. Foundations for Success: The Final Report of the National Mathematics Advisory Panel, U.S. Department of Education: Washington, D.C., 2008.

NOTES

Standard 1

Number Sense and Computation

CORE STANDARD

Number Sense and Computation

Counting to 20

Count objects and use objects, pictures and numerals to represent whole numbers up to 20. Find the number that is one more than or one less than any whole number up to 20. Recognize numbers from 10 to 20 and represent them as groups of tens and ones using objects, diagrams and numerals.

[Standard Indicators: K.1.1, K.1.2, K.1.4]

K.1.1 Count objects in a set and use objects, pictures and numerals to represent whole numbers to 20.

Example: Children match the oral counting numbers with each item in a set and give the last number counted as the quantity of the set. Students recognize five objects, five pictures, the word *five* and the numeral 5 as equivalent.

K.1.2 Find the number that is one more than or one less than any whole number up to 20.

Example: Play "One Less" Dominoes by placing a domino that represents one less than the last piece played and saying, "Five is one less than six," as they place their piece. Play "One More" Dominoes also.

K.1.3 Use correctly the words *one* and *many*; *none*, *some* and *all*; *more* and *less*; *most* and *least*; and *equal to*, *more than* and *less than*.

Example: Take some of the blocks out of the box, but not all of the blocks.

K.1.4 Show equivalent forms of whole numbers from 10 to 20 as groups of tens and ones using objects, diagrams and numerals.

Example: Using a two-column mat, put 10 counters on the left side and five counters on the right side. Identify the number of counters by saying 10 and five is 15. Vary the number of ones and repeat.

K.1.5 Model addition for numbers less than 10 by joining sets of objects and model subtraction by removing objects from sets.

Examples:

- Join a group of three cubes to a group of four cubes to make a group of seven cubes while saying, "7 = 4 + 3."
- From a pile of eight crayons, remove six crayons and tell how many crayons are left while saying, "8 – 6 = 2."

K.1.6 Record and organize information and answer questions about data using objects and pictures in context.

Example: Ask everyone in your class which color is his or her favorite. Color one box on grid paper for each child's response.

Standard 2

Algebra and Functions

K.2.1 Verbally describe mathematical relationships involving addition and subtraction situations for numbers less than 10.

Example: Using objects, verbally describe in an equation the joining of a group of three objects with one more object (e.g., $3 + 1 = 4$).

K.2.2 Create, extend and give the rule for simple patterns with numbers and shapes.

Example: Make a pattern with one square, one circle, one square, one circle, etc. Give a rule for the pattern and tell which shape comes next. Justify your choice.

Standard 3

Geometry and Measurement

CORE STANDARD

Geometry and Measurement

Comparing and Classifying Objects

Identify, describe, sort, compare and classify objects by shape, size, number of vertices and other attributes.

[Standard Indicator: K.3.1]

Comparing Measures

Make direct comparisons of the length and weight of objects and recognize which object is shorter, longer, taller, lighter or heavier.

[Standard Indicator: K.3.3]

K.3.1 Identify, describe, sort, compare and classify objects by shape, size, number of vertices and other attributes.

Example: Sort the tagboard shapes in the box into those that have straight sides and those that do not. Make up your own rule for sorting and sort the shapes in a different way.

K.3.2 Identify the positions of objects in space and use the terms *inside*, *outside*, *between*, *above*, *below*, *near*, *far*, *under*, *over*, *up*, *down*, *behind*, *in front of*, *next to*, *to the left of* and *to the right of.*

Example: Play "Simon Says" with words and motions to put the block inside, outside, above and below the box.

K.3.3 Make direct comparisons of the length and weight of objects and recognize which object is shorter, longer, taller, lighter or heavier.

Example: Line up pencils side by side, from shortest to longest, and explain the ordering used.

K.3.4 Identify concepts of time (*before*, *after*, *shorter*, *longer*, *morning*, *afternoon*, *evening*, *today*, *yesterday*, *tomorrow*, *week*, *month* and *year*).

Example: Tell about something you did yesterday and something you plan to do tomorrow while the teacher records the responses on a chart labeled *yesterday* and *tomorrow*.

PROCESS STANDARDS

Indiana's Academic Standards for Mathematics describe the key content of each grade level and course, and students must develop conceptual understanding of this content. The American Diploma Project noted that, "beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and 'mathematicizing' a problem" (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Round numbers to a specified place value.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurements in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning and problem-solving skills.
  - Elementary students should learn how to perform thoroughly the basic arithmetic operations independent of the use of a calculator.
  - The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

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National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston VA: author.

Standard 1

Number Sense and Computation

CORE STANDARD

Number Sense and Computation

Whole Numbers

Count, read, write and compare whole numbers up to 100. Represent numbers up to 100 as groups of tens and ones.

[Standard Indicators: 1.1.1, 1.1.4]

Addition and Subtraction

Model addition and subtraction using objects. Demonstrate fluency with addition facts and the corresponding subtraction facts for totals up to 20. Solve problems involving addition and subtraction.

[Standard Indicators: 1.1.5, 1.1.6]

1.1.1 Count, read, write, order, rename and compare whole numbers to at least 100.

Examples:

- Use a hundreds chart or manipulatives to compare numbers.
- Rename 87 as 50 + 37 and then rename in another way.

1.1.2 Name the number that is one more or one less than any number up to at least 100.

Example: Using a 0 – 99 chart and then without a chart name the number that is one more or one less than 79.

1.1.3 Match the ordinal numbers *first*, *second*, *third*, etc., with an ordered set to at least 10 items.

Example: Name the fifth child from the front of the line.

1.1.4 Show equivalent forms of whole numbers up to at least 100 as groups of tens and ones.

Example: Use base ten blocks to model 34 by using three longs and four ones, then 34 units, and finally by using two longs and 14 ones.

1.1.5 Solve problems involving addition and subtraction by using objects to model addition of numbers up to at least 100 (i.e., putting together, increasing) and by modeling the inverse operation of subtraction (i.e., taking away, comparing, finding the difference).

Examples:

- Using objects and a number line, show 75 equals 43 plus 32 more.
- Using objects and a number line, show the difference between 58 and 21.

1

1.1.6 Demonstrate fluency with addition facts and the corresponding subtraction facts for totals up to at least 20.

Example: Practice the "making tens" strategy by completing the following fact families:

$10 = 6 + 4$	$4 = 10 - 6$
$10 = \square + 6$	$6 = 10 - \square$
$\square + 6 = 10$	$10 - \square = 6$
$\square + 4 = 10$	$\square - 6 = 4$

1.1.7 Pose a question and collect and represent data using pictures or picture graphs to answer the question.

Example: Decide on a question, ask your classmates, record their responses and then make a picture graph of the results.

Standard 2

Algebra and Functions

1.2.1 Write and solve equations involving addition.

Example: $7 = \square + 3$.

1.2.2 Create, extend and give a rule for number patterns using addition.

Example: Given the number pattern 4, 8, 12… , tell the next number and explain how you determined the pattern to be followed.

1.2.3 Solve problems using the identity principle for addition and subtraction.

Example: Ben put five apples in a bag. When he got to school he realized the bag had a hole. He recounted the apples, and he had five. How many apples did Ben lose through the hole?

Standard 3

Geometry and Measurement

CORE STANDARD

Geometry and Measurement

Geometric Shapes

Identify, describe, compare, sort and draw triangles, rectangles, squares and circles.

[Standard Indicator: 1.3.1]

Linear Measurement

Estimate and measure the length of an object to the nearest inch and centimeter.

[Standard Indicator: 1.3.2]

1.3.1 **Identify, describe, compare, sort and draw triangles, rectangles, squares and circles in terms of their attributes (position, shape, size and number of vertices). Use simple plane shapes to compose a given shape.**

Examples:

- Draw shapes on the sidewalk and play a game jumping to the shape that is called.
- Compare squares and rectangles listing how they are alike and different.

1.3.2 **Estimate and measure the length of an object to the nearest inch and centimeter.**

Example: Locate items around the classroom that are close to one inch or one centimeter and list them on a t-chart. Measure to see if you chose items close to the correct unit.

1.3.3 **Give the value of a collection of pennies, nickels and dimes up to $1.00.**

Example: Play "Memory" with cards that have money amounts and pictures of coins. Collect matches and then tell your partner the value of the money represented.

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NOTES

Standard 1

Number Sense and Computation

CORE STANDARD

Number Sense and Computation

Place Value

Understand and use the relationship among whole numbers, including place value, to identify and compare numbers up to 1,000.

[Standard Indicators: 2.1.1, 2.1.2, 2.1.4]

Addition and Subtraction

Add and subtract whole numbers less than 1,000 using efficient methods. Understand and show the inverse relationship between addition and subtraction.

[Standard Indicator: 2.1.6]

2.1.1 Count, read, write, compare and plot on a number line whole numbers up to at least 1,000.

Examples:

- Use base ten blocks to model 234 by using two flats, three longs and four ones; then by using 23 longs and four ones; and finally by using 234 units.
- Which is more, 850 or 805? Justify your answer.

2.1.2 Count by ones, twos, fives, tens and hundreds to at least 1,000. Show the number that is 10 more or 10 less than any number from 10 through 90.

Example: Complete a chart with missing numbers and then tell what is 10 more or 10 less than any number given.

81	82	83	84	85	86	87		89	
	92			95					100
101								109	

2.1.3 Match the ordinal numbers *first*, *second*, *third*, etc., with an ordered set of at least 100 items.

Example: Given an alphabet chart, name the seventeenth letter in the alphabet.

2.1.4 Use words, models, standard form and expanded form to represent place value and to show equivalent forms of whole numbers up to at least 1,000 as groups of hundreds, tens and ones.

Example: Using base ten blocks and a place value mat show at least two different ways that you could model the number 57. Explain why your two models represent the same number.

2.1.5 Identify numbers as even or odd by placing that number of objects in two groups of the same size and recognizing that for even numbers no object will be left over and for odd numbers one object will be left over.

Example: Two children each pick an object from the pile of 18 objects and each place it in a pile they start. They continue picking objects and placing them in their own pile. After all of the objects have been picked and placed, count to see if each pile has the same number of objects.

2.1.6 Solve problems involving addition and subtraction of whole numbers less than 1,000 fluently using a standard algorithmic approach and show the inverse relationship between addition and subtraction.

Examples:

- The students wanted to know the total number of pencils they had remaining from their supply boxes. They worked in small groups and found that group one had 54 pencils, group two had 37 pencils and group three had 15 pencils. How many pencils did the class have? Explain how you found the answer.
- Their teacher said she had ordered a box of 500 pencils at the beginning of the year. How many pencils should still be in the box? Explain how you found the number of pencils.

2.1.7 Compare data from a single set or across sets of data to address a single question.

Example: Write a statement about the most and least favorite pets from your class and the class next door.

Standard 2

Algebra and Functions

2.2.1 Write equations to solve single and multi-step addition and subtraction word problems.

Example: You and a friend have collected 352 cans to recycle. You collected 201 cans. Write an equation to show the relationship between the numbers and to find how many cans your friend collected.

2.2.2 Create, extend and give a rule for number patterns using addition and subtraction.

Example: Find the next number in the sequence 101, 99, 97, 95… , and tell how you found the answer.

2.2.3 Show that the order in which two numbers are added (commutative property) and how the numbers are grouped in addition (associative property) will not change the sum. These properties can be used together to show that numbers can be added in any order.

Example: Add the numbers 5, 17 and 13 in this order. Now add them in the order 17, 13 and 5. Show that the results are the same.

Standard 3

Geometry and Measurement

CORE STANDARD

Geometry and Measurement

Common Shapes and Objects

Recognize, identify and describe attributes of common shapes and solids (e.g., the same size and type of shape; number of sides, edges and vertices; location in space).

[Standard Indicator: 2.3.1]

Linear Measurement

Measure lengths in standard units (e.g., inches, feet and yards) and metric units (e.g., centimeters and meters) and select appropriate units to estimate and measure lengths. Understand and use units of linear measurement and relationships within a particular system to solve problems.

[Standard Indicator: 2.3.3]

2.3.1 Recognize, identify and describe attributes of common shapes and solids (e.g., the size and type of shape; the two-dimensional faces of three-dimensional figures; the number of sides, edges and vertices; and location in space).

Examples:

- Describe the difference between a circle and a triangle using the terms *sides*, *edges* and *vertices*.
- Describe the difference between a cube and a sphere using the terms *sides*, *edges* and *vertices*.

2.3.2 Identify and draw congruent two-dimensional shapes in any position. Describe and compare properties of simple and compound figures composed of triangles, rectangles and squares.

Example: Given a shape on a geoboard, create a congruent shape somewhere else on the geoboard.

2.3.3 Measure length in standard units (inch, foot and yard) and metric units (centimeter and meter) and select appropriate units to estimate and measure lengths. Use the relationships between the units to express answers in different units. Use units of linear measurements and relationships within a particular system to solve problems.

Example: Estimate how many meter sticks would lie end to end across the classroom and then measure the length of your classroom to the nearest meter. Also report the result in centimeters.

2.3.4 Describe relationships of time (seconds in a minute, minutes in an hour, hours in a day, days in a week and days in a year) and tell time on an analog clock to the nearest five-minute intervals.

Examples:

- How long is your school day in hours?
- How many minutes do you work on math?
- Read an analog clock to help your teacher know when lunch and recess are over.

2.3.5 Find the value of a collection of pennies, nickels, dimes, quarters and dollars.

Example: You empty your bank and find the following: three pennies, four nickels and two dimes. Do you have enough money to buy a $0.50 pencil?

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Standard 1

Number Sense and Computation

CORE STANDARD

Number Sense and Computation

Place Value

Understand and use the relationship among whole numbers, including place value, to identify and compare numbers up to 10,000.

[Standard Indicators: 3.1.1, 3.1.4]

Interpreting and Modeling Fractions

Interpret and model fractions as parts of a whole, parts of a group, and points and distances on a number line. Solve problems that involve comparing and ordering fractions.

[Standard Indicators: 3.1.2, 3.1.3]

Addition and Subtraction

Understand and use standard algorithms for addition and subtraction.

[Standard Indicator: 3.1.5]

Multiplication and Division

Understand the meaning of multiplication and division of whole numbers and show the relationship between them.

[Standard Indicators: 3.1.6, 3.1.7]

3.1.1 Count, read, write, compare and plot on a number line whole numbers up to at least 10,000.

Examples:

- Show where 349 appears on the number line.
- Given another number, tell if 349 is less than or greater than this number.

3.1.2 Interpret and model fractions as parts of a whole, parts of a group, and points and distances on a number line for numbers less than, equal to or greater than one.

Examples:

- Shade $\frac{3}{4}$ of a given shape.
- Model $\frac{3}{4}$ of a collection of beans.
- Label $\frac{3}{4}$ on a number line that has $\frac{1}{4}$ increments marked.

3.1.3 **Compare and order fractions by using models, benchmark fractions, or common numerators or denominators.**

Example: Have students fold paper to make halves, fourths and eighths. Assign to groups of students specific amounts to color such as $\frac{1}{2}$ or $\frac{1}{4}$. Arrange the finished art on the wall from smallest to largest, showing how $\frac{2}{4} = \frac{1}{2}$.

3.1.4 **Use words, models, standard form and expanded form to represent place value and to show equivalent forms of whole numbers up to at least 10,000.**

Example: Convert among numbers written in words, standard form and expanded form, such as four hundred ninety-two = 492 = 400 + 90 + 2.

3

3.1.5 **Solve problems involving addition and subtraction of whole numbers fluently using a standard algorithmic approach.**

Example: Bob earned $547 in July and $568 in August. Bob told his friend he earned $1,103. Decide whether Bob is right or not and explain how you know.

3.1.6 **Represent the concept of multiplication of whole numbers with the following models: repeated addition, equal-sized groups, arrays, area models and equal "jumps" on a number line. Explain the result of multiplying by zero.**

Examples:

- Using base ten blocks, make an array of three rows of three blocks.
- Beginning at zero, make three hops of three to nine on the number line.
- Show the same amount as three groups of three items.

3.1.7 **Represent the concept of division of whole numbers with models as successive subtraction, partitioning, sharing and an inverse of multiplication. Show that division by zero is not possible.**

Examples:

- Start with 25 blocks and keep subtracting groups of five. How many groups of five can be made?
- Start with 25 blocks and find how many rows of five you can make.
- Start with 25 blocks and see how many groups of five you can make.
- Start with 25 blocks and make zero groups.
- Write the equation $\frac{25}{5} = 5$ in another way.

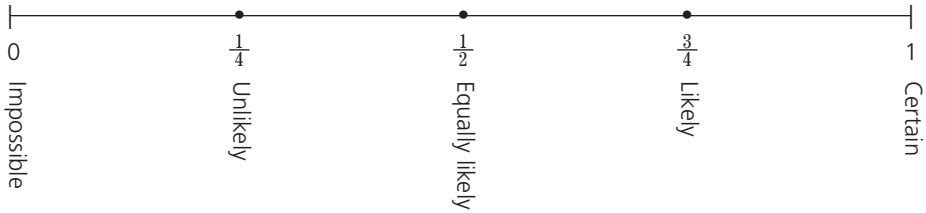
3.1.8 **Construct and analyze frequency tables and bar graphs from data, including data collected through observations, surveys and experiments.**

Example: Write a statement about the information displayed in the graph to express a conclusion about the information.

3.1.9 **Identify events on a continuum from impossible to unlikely, equally likely, likely or certain. Determine a simple probability in a context using pictures.**

Examples:

- Students will place the letter corresponding to the following events on the figure pictured below.



A. There are six cubes in a jar, and two are yellow. How likely are you to pull a yellow cube?

B. The probability that you will leave school before midnight.

C. The probability that a snowman in Indiana will stay in your backyard though the summer.

D. The probability that the sun will rise tomorrow.

- Flip a coin and then record the number of times the coin lands heads-up for 10 trials, then 20 trials.
- Spin a three-color spinner (red, yellow and green) and display the results for the number of times the spinner lands on a particular color.

Standard 2

Algebra and Functions

3.2.1 **Write and solve equations using (=) to show equivalence and use variables to express mathematical relationships involving multiplication.**

Example: If one ice cream sandwich costs 20 cents and two cost 40 cents, write the equation that will show how much n ice cream sandwiches cost.

3.2.2 **Create, extend and give a rule for number patterns by using multiplication.**

Example: Complete the following input-output table. Explain the rule you used to find the output.

Input	Output
5	50
2	20
7	

3.2.3 **Solve problems using the identity principle of multiplication.**

Example: Use arrays to model 1×5, 1×10, 1×3, 1×12 and then explain what happens whenever you multiply by one.

Standard 3

Geometry and Measurement

CORE STANDARD

Geometry and Measurement

Points and Lines

Identify, describe and draw points, lines and line segments.

[Standard Indicator: 3.3.2]

Length, Weight and Unit Conversions

Choose and use appropriate units and tools to estimate and measure length and weight. Use the relationship between the units to express answers in different units.

[Standard Indicator: 3.3.5]

3.3.1 Identify angles that are right angles and other angles that are greater than or less than a right angle.

Example: Use the corner of a piece of paper as a right angle finder to search for right angles in the classroom.

3.3.2 Identify, describe and draw points, lines and line segments and use these terms when describing two-dimensional shapes.

Example: Draw a line segment $\overline{AB}$ that is 2 cm long.

3.3.3 Identify and draw lines of symmetry in geometric shapes and recognize symmetrical shapes in the environment.

Example: Use pencil and paper or a drawing program to draw all the lines of symmetry in a square.

3.3.4 Find the perimeter of polygons.

Example: Measure your desk in centimeters and then find the perimeter.

3.3.5 Choose and use appropriate units and tools to estimate and measure length and weight. Estimate and measure length to a quarter-inch, weight in pounds and kilograms, and read temperature in Celsius and Fahrenheit. Select appropriate units for the given situation. Use the relationship between the units to express answers in different units.

Examples:

- Estimate and then measure the weight of your book bag in pounds and in ounces.
- Estimate and then read the temperature on a thermometer in degrees Fahrenheit and degrees Celsius.

3.3.6 Using an analog clock, tell time to the nearest minute.

Example: If you have to be to school at 8:00 a.m. and the car ride is 15 minutes, when do you need to leave for school?

PROCESS STANDARDS

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- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
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- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Round numbers to a specified place value.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
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- Use properties of numbers and operations to perform mental computation.
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Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
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Standard 1

Number Sense and Computation

CORE STANDARD

Number Sense and Computation

Place Value

Understand and use the relationship among whole numbers, including place value, to identify and compare numbers. Interpret and model decimals as parts of a whole, parts of a group, and points and distances on a number line.

[Standard Indicators: 4.1.1, 4.1.3, 4.1.4]

Multiplication and Division Facts

Demonstrate fluency with multiplication facts for numbers up to 10 and related division facts.

[Standard Indicator: 4.1.5]

Multiplying Whole Numbers

Multiply numbers up to 100 by single-digit numbers and two-digit numbers.

[Standard Indicator: 4.1.6]

Addition and Subtraction of Fractions

Model addition and subtraction of simple fractions.

[Standard Indicators: 4.1.2, 4.1.7]

4.1.1 Count, read, write, compare and plot whole numbers using words, models, number lines and expanded form.

Example: Plot the number 980,000 on a number line labeled in increments of 100,000s.

4.1.2 Find equivalent fractions and then use them to compare and order whole numbers and fractions using the symbols for less than (<), equals (=) and greater than (>).

Examples:

- Find three equivalent fractions for $\frac{1}{4}$.
- List from least to greatest: $\frac{7}{8}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{1}{3}$.

4.1.3 Solve problems involving decimals to hundredths.

- Interpret and model decimals as parts of a whole, parts of a group, and points and distances on a number line.
- Use benchmarks (well-known numbers used in meaningful points for comparison) to compare decimals between 0 and 1.0.
- Write decimals as fractions.

  Examples:

  - Model 0.4 on a ten-by-ten grid.
  - Place 0.5 and 0.8 on a number line marked in tenths.
  - Is 0.4 closer to 0 or to 1?
  - Write 0.4 as a fraction.

4

4.1.4 Use words, models, standard form and expanded form to represent place value of decimal numbers to hundredths.

Example: Explain how you know that 0.4 is equal to 0.40.

4.1.5 Demonstrate fluency with multiplication facts for numbers up to at least 10 and the related division facts. Identify factors of whole numbers and multiples of whole numbers to 10.

Examples:

- Complete basic facts like $9 \times 4 = \square$ and $35 \div 7 = \square$ quickly and accurately.
- List all the factors for 36.
- Find the common multiples of four and six.

4.1.6 Solve problems fluently by using multiplication of two-digit numbers by a single-digit number and by using multiplication of two-digit numbers by other two-digit numbers. Use a standard algorithmic approach.

Example: If there are 24 boxes with 36 pencils in each, what is the total number of pencils? Estimate and then compute the product.

4.1.7 Model addition and subtraction of simple fractions.

Example: Using pattern blocks show that if the yellow hexagon equals one whole, which part of the hexagon represents $1 - \frac{1}{3}$?

4.1.8 Construct and analyze line plots. Given a set of data or a graph, describe the distribution of the data using median, range or mode.

Example: Display the following amounts of seconds from your science experiment: 16, 22, 16, 9, 11, 16 and 11 as a line plot. Identify the median, range and mode of your data.

4.1.9 List all the possible outcomes of a given situation or event. Represent the probability of a given outcome using a picture or other graphic.

Example: The Circle Snack Shop has three flavors of ice cream: vanilla, chocolate and strawberry. The ice cream can be served in a sugar cone, waffle cone or dish. List all the possible combinations of flavors of ice cream and how they are served.

Standard 2

Algebra and Functions

4.2.1 Write and solve equations with the symbol for equals to (=) to show equivalence and use the symbol for equals to (=) with variables to express mathematical relationships involving multiplication and division.

Examples:

- Find the number n which satisfies the equation $n \times 7 = 42$.
- Solve the equation $n \times 6 = 9 \times 4$.

4.2.2 Create, extend and give a rule for number patterns using multiplication, division, non-numeric growing and non-numeric repeating patterns.

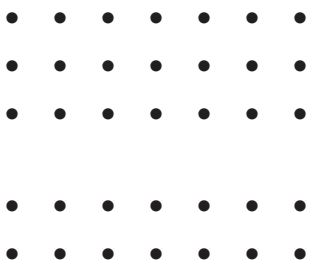
Example: Given a verbal description, create a different representation of a pattern or sequence.

4.2.3 Show that the order in which two numbers are multiplied (commutative property) and how numbers are grouped in multiplication (associative property) will not change the product. Use these properties together to show that numbers can be multiplied in any order.

Example: Draw or build arrays to show that three rows of eight objects use the same number of objects as eight rows of three objects.

4.2.4 Use the distributive property in expressions involving multiplication.

Example: Use the array model to show that $5 \times 7 = (3 \times 7) + (2 \times 7)$.



Standard 3

Geometry and Measurement

CORE STANDARD

Geometry and Measurement

Angles and Lines

Identify, describe and draw parallel and perpendicular lines and right, acute, obtuse and straight angles.

[Standard Indicators: 4.3.1, 4.3.2]

Rectangles

Find and use the perimeter and area of rectangles, including squares.

[Standard Indicator: 4.3.5]

4

4.3.1 Identify, describe and draw pairs of parallel lines, perpendicular lines and non-perpendicular intersecting lines using appropriate mathematical tools and technology.

Example: Using their arms, students will model parallel lines, perpendicular lines and non-perpendicular intersecting lines.

4.3.2 Identify, describe and draw right angles, acute angles, obtuse angles, straight angles and rays using appropriate tools and technology.

Example: Draw two rays that meet to form each of the angles listed above.

4.3.3 Identify shapes that have reflectional and rotational symmetry.

Example: Make a masking tape frame on the floor for a cardboard equilateral triangle. Mark one corner of an equilateral triangle that fits the frame and the corresponding corner on the frame. Rotate the triangle clockwise until it again fits the frame, rotating about what could be called the center of the triangle. Continue to rotate the triangle until it fits again, and continue until the triangle returns to its original position inside the frame. Count how many times the shape will fit in the frame until it returns to its original position. Discuss this triangle as having rotational symmetry of the order of three.

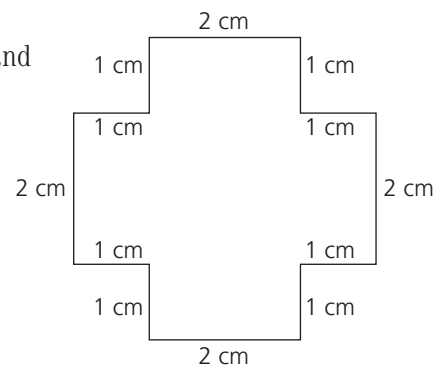
4.3.4 Measure and draw line segments to the nearest eighth-inch and millimeter.

Example: Measure across the face of a nickel to the nearest millimeter.

4.3.5 Develop and use formulas for finding the perimeter and area of rectangles (including squares) by using appropriate strategies (i.e., decomposing shapes), tools and units of measure.

Examples:

- Measure the length and width of a basketball court and find its area in suitable units.
- Find the area and perimeter of the following shape.



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Standard 1

Number Sense and Computation

CORE STANDARD

Number Sense and Computation

Multiplication and Division of Whole Numbers

Understand and use standard algorithms for multiplication and division of whole numbers.

[Standard Indicator: 5.1.5]

Comparing Fractions and Decimals

Compare fractions and decimals.

[Standard Indicators: 5.1.1, 5.1.2]

Addition and Subtraction of Fractions and Decimals

Understand and perform addition and subtraction with fractions, including fractions with different denominators and mixed numbers. Add and subtract decimals, including money in decimal notation.

[Standard Indicator: 5.1.6]

5.1.1 Count, read, write, compare and plot on a number line decimals to thousandths using words, models and expanded form.

Example: Write the number 198.536 in words.

5.1.2 Compare and order fractions and decimals to thousandths by using the symbols for less than (<), equal to (=) and greater than (>).

Example: Write from the least to the greatest using the correct symbols: 0.5, 0.26, 1.2 and 0.008.

5.1.3 Identify and explain prime and composite numbers.

Example: List the first five prime counting numbers. Tell why they are prime numbers.

5.1.4 Use words, models, standard form and expanded form to represent place value of decimal numbers to thousandths.

Example: Explain how you know that 0.005 is less than 0.050.

5.1.5 Solve problems involving multiplication and division of whole numbers fluently using a standard algorithmic approach and explain how to treat the remainders in division.

Example: Twenty-six students are going on a field study using the school vans. How many vans are needed if each van holds six students and a driver?

5.1.6 Solve problems using a standard algorithmic approach involving addition and subtraction of:

- decimals, including money;
- fractions, including fractions with different denominators; and
- mixed numbers.

Examples:

- How much money will you have coming home if you go to the store with $3.00 and you buy popcorn for $0.95 and a drink for $1.25?
- The cheer squad has $2\frac{1}{4}$ yards of blue fabric, but they need 5 yards to make all the banners wanted for the game. How much fabric is still needed?

5.1.7 Solve problems involving the multiplication of fractions using a standard algorithmic approach. Explain the relationship of the product relative to the factors.

Example: After the party there is $\frac{1}{2}$ of a pizza left. If you give $\frac{1}{3}$ of the leftover pizza to your brother, how much of the whole pizza will he get? Explain why the product is smaller than the factors.

5

5.1.8 Construct and analyze line graphs and double-bar graphs from data, including data collected through observations, surveys and experiments.

Example: Complete a line graph to show the changes in temperature over a month.

5.1.9 Perform simple experiments to gather data from a large number of trials and use data from experiments to predict the chance of future outcomes.

Example: Using a three-color spinner with red, yellow and blue:

- Tally the result for 10 spins and then for 30 spins.
- Combine your results with your class and compare results.
- Describe what happens as the number of trials increases.
- Can you determine on which color the next spin will land?
- Can you tell the likelihood that the next spin will land on yellow?

Standard 2

Algebra and Functions

CORE STANDARD

Algebra and Functions

Variables

Evaluate simple algebraic expressions.

[Standard Indicator: 5.2.1]

Coordinate Grids

Use two-dimensional coordinate grids to represent points in the first quadrant that fit linear equations and then draw the line determined by the points.

[Standard Indicator: 5.2.2]

5.2.1 Write and evaluate simple algebraic expressions.

Example: Write the equation to represent how many cookies you originally had if you put three cookies in each of five bags and you had two extra. What is the value if there are four cookies in each bag, and you still had two extra?

5.2.2 Use two-dimensional coordinate grids to represent points in the first quadrant that fit linear equations and then draw the line determined by the points.

Example: Plot the points (3, 1), (6, 2) and (9, 3). Connect the points and describe what you notice.

Standard 3

Geometry and Measurement

CORE STANDARD

Geometry and Measurement

Polygons

Measure angles; describe angles in degrees; and identify, classify and draw polygons and triangles.

[Standard Indicators: 5.3.1, 5.3.2]

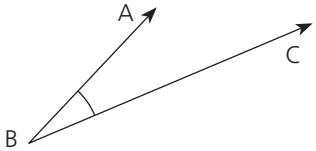
Shapes and Solids

Find and use the perimeter and area of triangles, parallelograms and trapezoids, and the surface area and volume of rectangular prisms.

[Standard Indicators: 5.3.5, 5.3.6]

5.3.1 Measure angles and describe angles in degrees.

Example: Measure the following angle.



5.3.2 Identify, classify and draw polygons and triangles (i.e., equilateral, isosceles, scalene, right, acute and obtuse triangles).

Examples:

- Draw as many different types of quadrilaterals as possible.
- Draw an isosceles right triangle.

5.3.3 Describe the attributes (such as number of edges, vertices and number of faces) of solids, including cubes, pyramids and cylinders.

Example: Complete the following table.

Solid	Faces	Edges	Vertices
cube	6	12	8
square pyramid		8	5
cylinder	3	0	0
rectangular prism			

5.3.4 Identify and describe, using words and pictures, the following transformations: reflections, rotations and translations. Use this knowledge to design and analyze simple tilings and tessellations.

Example: Using tiles or grid paper create a design that does not have any gaps or holes.

5.3.5 **Develop and use the formulas for the perimeter and area of triangles, parallelograms and trapezoids using appropriate units for measures. Find the area of complex shapes by dividing them into basic shapes.**

Example: You want to carpet a square room whose sides are 17 feet. You are not going to carpet the area near the fireplace that is 6 feet long and 4 feet wide. What is the area to be carpeted?

5.3.6 **Develop and use the formulas for the surface area and volume of rectangular prisms using appropriate units for measures.**

Example: Find the surface area and volume of a rectangular box with a length of 30 cm, width of 15 cm and a height of 10 cm.

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Standard 1

Number Sense and Computation

CORE STANDARD

Number Sense and Computation

Positive and Negative Numbers

Understand and apply the concept of positive and negative numbers. Add, subtract, multiply and divide positive and negative integers. Represent negative numbers and computation with negative numbers on a number line.

[Standard Indicators: 6.1.1, 6.1.5]

Percent Representations

Use percents to represent parts of a whole. Represent numbers as fractions, decimals and percents.

[Standard Indicators: 6.1.3, 6.1.4]

Multiplication and Division of Fractions and Decimals

Understand and perform multiplication and division with positive decimals and fractions.

[Standard Indicator: 6.1.6]

Ratio and Rate

Solve simple ratio and rate problems using multiplication and division.

[Standard Indicators: 6.1.7, 6.1.8, 6.1.9]

6.1.1 Compare, order and represent on a number line positive and negative integers, fractions, decimals to hundredths and mixed numbers.

Example: Find the positions on a number line of 3.56, -2.5, $1\frac{5}{6}$, -4 and $-\frac{3}{4}$.

6.1.2 Interpret the absolute value of a number as the distance from zero on a number line. Find the absolute value of real numbers and know that the distance between two numbers on the number line is the absolute value of their difference.

Example: Use a number line to explain the absolute values of -3 and 7.

6.1.3 Use percents to represent parts of a whole. Find the percents part of a whole.

Example: Draw a circle and shade 45 percent of it.

6.1.4 Recognize commonly used fractions, decimals, percents and all of their equivalents and convert between any two representations of non-negative rational numbers without the use of a calculator.

Example: Know that $\frac{1}{3} = 0.333\ldots$, $\frac{1}{2} = 0.5$, $\frac{2}{5} = 0.4$, etc.

6.1.5 Solve problems involving addition, subtraction, multiplication and division of integers and represent computation with integers on a number line. Describe the effect of operations with numbers less than zero.

Example: $17 + -4 = \square$; $-8 - 5 = \square$; $3(-6) = \square$; $-12 \div -2 = \square$.

6.1.6 Solve problems involving addition, subtraction, multiplication and division of positive fractions and decimals. Explain why a particular operation was used for a given situation.

Examples:

- You want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide. How far from each edge should you place the bar? Explain your method.
- Share $7.25 equally among five people.

6.1.7 Interpret ratios, model ratios and use ratios to show the relative sizes of two quantities.

- Use the notations: $\frac{a}{b}$, *a to b* and *a*:*b*.
- Write equivalent ratios.
- Express a ratio in its simplest form.
- Find the ratio of two given quantities.

Example: A car moving at a constant speed travels 130 miles in 2 hours. Write the ratio of distance to time as a fraction in reduced form.

6.1.8 Recognize proportional relationships and solve problems involving proportional relationships. Find the missing term in a pair of equivalent ratios. Find one quantity given the other quantity and their ratio to each other.

Example: Sam made eight out of 24 free throws. Assuming Sam's success rate continues, find how many free throws he will make with 60 attempts.

6.1.9 Solve simple percent, ratio and proportion problems, including problems involving discounts at sales, interest earned and tips.

Example: In a sale, everything is reduced by 20 percent. Find the sale price of a shirt whose pre-sale price was $30.

Standard 2

Algebra and Functions

CORE STANDARD

Algebra and Functions

Linear Equations

Write and solve one-step equations and inequalities in one variable.

[Standard Indicators: 6.2.1, 6.2.3]

Linear Functions

Use equations and graphs of linear functions to represent a given situation.

[Standard Indicators: 6.2.4, 6.2.5]

6.2.1 Write and solve one-step linear equations and inequalities in one variable.

Example: The area of a rectangle is 143 cm^2 and the length is 13 cm. Write and solve an equation to find the width of the rectangle. Describe how you will check to be sure that your answer is correct.

6.2.2 Write and use formulas with up to three variables to solve problems.

Example: You have P dollars in a bank that gives r percent simple interest per year. Write a formula for the amount of interest you will receive in one year. Use the formula to find the amount of interest on \$80 at 6 percent per year for one year.

6.2.3 Apply the correct order of operations and the properties of real numbers (i.e., identity, inverse, commutative, associative and distributive properties) to evaluate numerical expressions, including those that use grouping symbols like parentheses. Justify each step in the process.

Example: Simplify $3(4 - 1) + 2$. Explain your method.

6.2.4 Identify and graph ordered pairs in all four quadrants of the coordinate plane.

Example: Plot the points (3, -1), (-6, 2) and (9, -3). Describe the results.

6.2.5 Solve problems involving linear functions with integer values. Create a table and graph the resulting ordered pairs of integers on a grid. Look for patterns in how a change in one variable relates to a change in the second variable and write an equation that models the relationship.

Example: A plant is 3 cm high the first time you measure it (on Day 0). Each day after that, the plant grows by 2 cm. Write an equation connecting the height and the number of the day. Draw its graph.

Standard 3

Geometry and Measurement

CORE STANDARD

Geometry and Measurement

Angles and Polygons

Use properties of complementary, supplementary and vertical angles; properties of triangles; and properties of quadrilaterals to find missing angles.

[Standard Indicators: 6.3.1, 6.3.2]

Shapes and Solids

Find and use the circumference and area of circles and the surface area of right prisms and cylinders.

[Standard Indicators: 6.3.3, 6.3.5]

6.3.1 Identify, draw and use the properties of vertical, adjacent, complementary and supplementary angles and the properties of triangles and quadrilaterals to solve problems involving the measure of an unknown angle.

Example: Draw two parallel lines with another line across them. Identify all pairs of supplementary angles.

6.3.2 Recognize that the sum of the interior angles of any triangle is 180° and that the sum of the interior angles of any quadrilateral is 360°. Use this information to solve problems.

Example: Find the size of the third angle of a triangle with angles of 73° and 49°.

6.3.3 Develop and use the formula for the circumference of a circle and the formula for the area of a circle.

Example: Measure the diameter and circumference of several circular objects. (Use string to find the circumference.) With a calculator, divide each circumference by its diameter. What do you notice about the results?

6.3.4 Recognize that real-world measurements are approximations. Identify appropriate instruments and units for a given measurement situation, taking into account the precision of the measurement desired.

Example: A coach is using a wristwatch to measure how fast Carlos and Tyler run a 50-yard dash. The wristwatch displays only hours and minutes. Explain why this wristwatch is an inappropriate instrument.

6.3.5 Develop and use the formula for the surface area of a cylinder and volume of a cylinder and find the surface area and volume of three-dimensional objects built from rectangular solids and cylinders.

Example: Find the surface area of a cylindrical can 15 cm high and with a diameter of 8 cm.

Standard 4

Data Analysis and Probability

6.4.1 **Construct and analyze circle graphs and stem-and-leaf plots.**

Example: Display the following data in a stem-and-leaf plot: $126, $118, $100, $98, $95, $138, $101 and $116. Write a sentence that summarizes the data. Explain if a circle graph would be a useful way to display this data.

6.4.2 **Choose the appropriate display for a set of data from bar graphs, line graphs, circle graphs and stem-and-leaf plots. Justify the choice of data display.**

Example: Sarah is investigating the price of a gallon of milk in 15 different supermarkets for a school project. Determine whether a bar graph, line graph, circle graph or a stem-and-leaf plot is the best way to display the data. Explain how the type of display you chose is useful in displaying the data.

6.4.3 **Compare the mean, median and mode for a set of data and explain which measure is most appropriate in a given context.**

Example: The following numbers represent the number of touchdown passes completed by seven different quarterbacks during a three-year period: 74, 42, 36, 31, 27, 27, 27. Which statistical summary (mean, median or mode) is most representative of the data? Explain. If an eighth quarterback who completed 90 passes is added to the data, which measure would not be affected?

6.4.4 **Solve problems involving probability as a measure of chance and verify that the probabilities computed are reasonable.**

Example: There are three blue, five green and 12 red marbles in a bag. You pick one at random. Write the probability of picking a green marble as a fraction, decimal and percent.

6.4.5 **Recognize and represent probabilities as ratios, measures of relative frequency, decimals between 0 and 1, and percents between 0 and 100.**

Example: The probability that the Colts will win the Super Bowl next year is 0.85. Express this probability as a ratio and a percent.

PROCESS STANDARDS

Indiana's Academic Standards for Mathematics describe the key content of each grade level and course, and students must develop conceptual understanding of this content. The American Diploma Project noted that, "beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and 'mathematicizing' a problem" (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Round numbers to a specified place value.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurements in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning and problem-solving skills.
  - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
  - The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

American Diploma Project (2004). *Ready or Not: Creating a High School Diploma that Counts.* Washington, D.C.: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston VA: author.

NOTES

Standard 1

Number Sense and Computation

CORE STANDARD

Number Sense and Computation

Exponents

Use whole number exponents for repeated multiplication. Use scientific notation for large numbers.

[Standard Indicators: 7.1.1, 7.1.2]

Proportions and Percents

Use proportions and percents to solve application problems involving the increase of a quantity and the decrease of a quantity. Solve problems involving percents, ratios, rates and similar triangles.

[Standard Indicators: 7.1.8, 7.1.9]

Multiplication and Division with Fractions and Decimals

Understand and perform multiplication and division with negative decimals and fractions.

[Standard Indicator: 7.1.7]

7.1.1 Read, write, compare and solve problems using whole numbers in scientific notation.

Example: Write 300,000 in scientific notation.

7.1.2 Recognize and compute whole number powers of positive integers.

Example: $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = \square$

7.1.3 Recognize the prime factors of a number and find the prime factorization of whole numbers. Write the results using exponents.

Example: $24 = 2 \times 2 \times 2 \times 3 = \square$

7.1.4 Recognize or use prime and composite numbers to solve problems.

Example: A counterexample is an example showing that a statement is not true. Find a counterexample for the statement "All even numbers are composite numbers" and explain how you know it is a counterexample.

7.1.5 Recognize and use the inverse relationship between squaring and finding the square root of a perfect square integer.

Example: Find the area of a square with 5-cm sides and find the side of a square whose area is 144 square cm.

7.1.6 Identify, write, rename, compare and order rational and common irrational numbers and plot them on a number line.

Example: Write in order from smallest to largest: -2, -2π, $-\sqrt{2}$, $-2\sqrt{2}$.

7.1.7 Solve problems that involve multiplication and division with integers, fractions, decimals and combinations of the four operations.

Example: The temperature one day is 5°. It then falls by 3° each day for four days and, after that, rises by 2° each day for three days. What is the temperature on the last day? Explain your method.

7.1.8 Solve problems involving percents.

- Find the whole given a part and the percent.
- Find the percent increase or decrease.

Example: The population of a country was 36 million in 1990 and it rose to 41.4 million during the 1990s. What was the percent increase in the population?

7.1.9 Solve problems involving ratios and proportions.

- Given their ratio, express one quantity as a fraction of another and vice versa.
- Given their ratio, find how many times one quantity is as large as another and vice versa.
- Given the two quantities, express one quantity as a fraction of another.
- In a given ratio, find the whole, or one part, when a whole is divided into parts.
- Solve problems involving two pairs of equivalent ratios.

Example: On a survey of females in an exercise class, 12 out of the 20 females in the class indicated they prefer to exercise in the morning. What percent of the females in the class prefer to exercise in the morning?

Standard 2

Algebra and Functions

CORE STANDARD

Algebra and Functions

Expressions

Evaluate numerical expressions and simplify algebraic expressions involving rational and irrational numbers.

[Standard Indicators: 7.2.1, 7.2.3]

Linear Equations

Write and solve two-step equations and inequalities in one variable.

[Standard Indicators: 7.2.1, 7.2.2]

Graphs of Lines

Find the slope of a line from its graph and relate the slope of a line to similar triangles. Draw the graph of a line given either its slope and one point on the line or two points on the line. Graph proportional relationships and identify the unit rate as the slope of the related line.

[Standard Indicators: 7.2.5, 7.2.6, 7.2.7]

7

7.2.1 **Use variables and appropriate operations to write an expression, equation or inequality that represents a verbal description.**

Example: Using symbols, write the following inequality: five less than twice the number is greater than 42.

7.2.2 **Write and solve two-step linear equations and inequalities in one variable.**

Example: Solve the equation $4x - 7 = 12$ and check your answer in the original equation.

7.2.3 **Evaluate numerical expressions and simplify algebraic expressions involving rational and irrational numbers.**

Example: Simplify $3(4x + 5x - 1) + 2(x + 3)$. Explain each step you take.

7.2.4 **Solve an equation or formula with two variables for a particular variable.**

Example: Solve the formula $C = 2\pi r$ for r.

7.2.5 **Find the slope of a line from its graph and relate the slope of a line to similar triangles.**

Example: Draw the graph of $y = 2x - 1$. Choose two points on the graph and divide the change in y-value by the change in x-value. Repeat this for other pairs of points on the graph. What do you notice?

7.2.6 **Draw the graph of a line given either its slope and one point on the line or two points on the line.**

Example: Draw the graph of the equation with slope of 3 and passing through the point (0, -2).

7.2.7 Identify situations that involve proportional relationships, draw graphs representing these situations and recognize that these situations are described by a linear function in the form $y = mx$, where the unit rate m is the slope of the line.

Example: At a ski resort, one of the slopes rises 8 feet vertically for every 48-foot run. The second slope rises 12 feet vertically for every 72-foot run. Compare the steepness of the two slopes. Which is steeper?

Standard 3

Geometry and Measurement

CORE STANDARD

Geometry and Measurement

Transformations

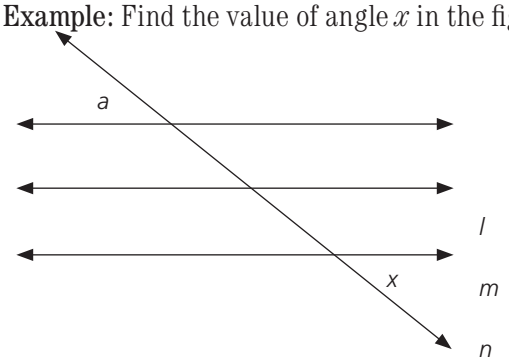
Identify and use the following transformations: translations, rotations and reflections.

[Standard Indicator: 7.3.2]

7

7.3.1 Identify and use basic properties of angles formed by transversals intersecting parallel lines.

Example: Find the value of angle x in the figure below if $m\angle a = 30°$



7.3.2 Identify, describe and use transformations (translations, rotations, reflections and simple compositions of these transformations) to solve problems.

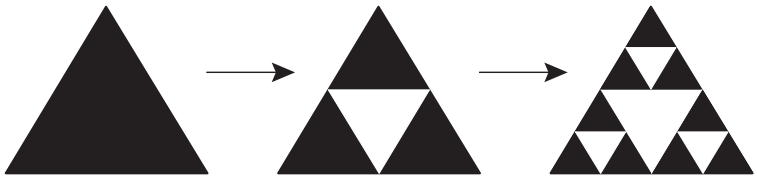
Example: Draw a triangle with vertices (2, 3), (5, 3) and (2, 7). Translate (slide) the triangle so that the vertex (2, 3) moves to (0, 0). Find the new coordinates of the other two vertices.

7.3.3 Draw two-dimensional patterns (nets) for three-dimensional objects like right prisms, pyramids, cylinders and cones.

Example: Draw a rectangle and two circles that will fit together to make a cylinder.

7.3.4 Recognize, describe or extend geometric patterns using tables, graphs, words or symbols.

Example: Look at the three triangles below. What fraction of each triangle is not shaded? Do you see a pattern? Use the pattern to predict the fraction of the triangle you would NOT shade in the fourth iteration of the triangle. Confirm your prediction and explain.



7.3.5 Identify, describe and construct similarity relationships and solve problems involving similarity (including similar triangles). Scale drawings by using proportional reasoning.

Example: At a certain time, the shadow of your school building is 36 feet long. At the same time, the shadow of a yardstick held vertically is 4 feet long. What is the height of the school building?

7.3.6 Solve simple problems involving distance, speed and time.

- Understand concepts of speed and average speed.
- Understand the relationships among distance, time and speed.
- Find speed, distance or time given the other two quantities.
- Write speed in different units (km/h, m/s, cm/s, mi/hr, ft/sec).
- Solve simple problems involving speed and average speed.

Examples:

- Find how long an airplane flying at 900 kilometers per hour takes to travel 1,350 kilometers.
- A cheetah can run 120 kilometers per hour for a short time. Give this speed in meters per second.
- Sarah drove 145 miles from Elkhart to Muncie in 2 hours and 45 minutes. Find Sarah's average speed in miles per hour.

Standard 4

Data Analysis and Probability

CORE STANDARD

Data Analysis and Probability

Making Estimates and Data Displays

Use proportions to make estimates about a population based on a sample. Create, analyze and interpret data sets in multiple ways using bar graphs, frequency tables, line plots, histograms and circle graphs.

[Standard Indicators: 7.4.1, 7.4.2]

Theoretical Probability

Understand that when all outcomes are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event may occur. Use theoretical probability and proportions to make predictions.

[Standard Indicator: 7.4.5]

7.4.1 **Create, analyze and interpret data sets in multiple ways using bar graphs, frequency tables, line plots, histograms and circle graphs. Justify the choice of data display.**

Example: The students will count the value of the change in their pockets. Use the amounts collected from the class to construct a histogram. Describe the shape of the distribution.

7.4.2 **Make predictions from statistical data and use proportions to make estimates about a population based on a sample.**

Example: Record the temperature and weather conditions (sunny, cloudy or rainy) at 1 p.m. each day for two weeks. In the third week, use your results to predict the temperature from the weather conditions.

7.4.3 **Describe how additional data, particularly outliers, added to a data set may affect the mean, median and mode.**

Example: You measure the heights of the students in your grade on a day when the basketball team is playing an away game. Later you measure the players on the team and include them in your data. What kind of effect will including the team have on the mean, median and mode? Explain your answer.

7.4.4 **Analyze data displays, including ways that they can be misleading. Analyze ways in which the wording of questions can influence survey results.**

Example: A company displays a bar graph of company's sales that suggests sales have more than doubled since last year. Upon analyzing the graph, you notice that sales have in fact increased from $5.5 million to $6.2 million. Explain how the company may have used the graph to suggest that sales doubled.

7.4.5 **Understand that when all outcomes are equally likely, the theoretical probability of an event is the fraction of outcomes in which the event may occur. Use theoretical probability and proportions to make predictions.**

Example: The weather forecast says that the probability of rain today is 0.3. What is the probability that it will not rain?

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References

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Standard 1

Number Sense and Computation

CORE STANDARD

Number Sense and Computation

Integer Exponents

Use the laws of integer exponents and evaluate expressions with negative integer exponents. Use scientific notation for small numbers.

[Standard Indicators: 8.1.1, 8.1.2, 8.1.3]

Square Roots

Use irrational numbers. Calculate square roots. Use the inverse relationship between squares and square roots.

[Standard Indicators: 8.1.4, 8.1.5]

8.1.1 Interpret calculator or computer displays of numbers given in scientific notation and read, write, compare and solve problems using decimals in scientific notation.

Example: Polaris is 4.077×10^{15} km from Earth. There are 9.461×10^{12} km in a light year. Find the distance from Earth to Polaris in light years.

8.1.2 Recognize positive integer powers as repeated multiplication. Recognize negative integer powers as repeated division or multiplication by the multiplicative inverse.

Example: Write 2^{-3} as a fraction.

8.1.3 Use the laws of exponents for integer exponents and evaluate expressions with negative integer exponents.

Examples:

- Write $2^2 \times 2^3$ as $(2 \times 2)\ (2 \times 2 \times 2)$ and then as a single power of 2. Explain your process.
- Evaluate $\frac{6^{-3}}{6^2}$. Explain your process.

8.1.4 Identify, compare and order irrational numbers.

Example: Estimate the square root of $\sqrt{18}$ to the nearest tenth. Plot the square root on a number line.

8.1.5 Calculate square roots of perfect squares, estimate square roots of numbers less than 1,000, and use the inverse relationship between squares and square roots.

Example: Explain how you can find the length of the hypotenuse of a right triangle with legs that measure 5 cm and 12 cm.

8.1.6 Solve percent, ratio and proportion problems.

- Find average rates.
- Express one quantity as a percent of another.
- Compare two quantities by percent.
- Use percents greater than 100 percent.
- Increase or decrease a quantity by a given percent.
- Find the original amount for a given percent increase or decrease.
- Solve problems involving percents, ratios and proportions.
- Solve problems involving simple and compound interest.

  Example: You leave $100 in each of three bank accounts paying 5 percent interest per year. One account pays simple interest, one pays interest compounded annually, and the third pays interest compounded quarterly. Use a spreadsheet to find the amount of money in each account after one, two, three, 10 and 20 years. Compare the results in the three accounts and explain how compounding affects the balance in each account.

Standard 2

Algebra and Functions

CORE STANDARD

Algebra and Functions

Solving Equations and Inequalities

Write and solve multi-step equations and inequalities in one variable.

[Standard Indicator: 8.2.1]

Linear Functions

Use linear functions and linear equations to represent, analyze and solve problems. Translate among tables, equations, verbal expressions and graphs.

[Standard Indicators: 8.2.4, 8.2.5, 8.2.6, 8.2.8]

8.2.1 Write and solve linear equations and inequalities, interpret the solution or solutions in their context, and verify the reasonableness of the results.

Example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, solve it and check that your answer is reasonable.

8.2.2 Solve equations and formulas for a specified variable.

Example: Solve $5(x + 2) = 10 + 5$

8.2.3 **Simplify algebraic expressions involving powers.**

Example: Expand $3(2x + 5)^2$

8.2.4 **Identify and graph linear functions and identify lines with positive and negative slopes.**

Example: Draw the graphs of $y = 2x - 1$, $y = 3x - 1$, $y = -2x - 1$ and $y = -3x - 1$. Find the slope of each graph. What do you notice?

8.2.5 **Find the slope of a linear function given the equation. Write the equation of a line given the slope and any point on the line.**

Example: Write an equation of the line with slope 2 and containing the point (1, -2). What is the y-intercept for this line?

8.2.6 **Translate among tables, equations, verbal expressions and graphs of linear functions and recognize in ($y = mx + b$) that m is the rate of change and b is the vertical intercept of the graph.**

Example: Write a formula for the perimeter of a square as a function of its side length. Construct a table of values for this function. Draw the graph of this function.

8.2.7 **Identify functions as linear or nonlinear and contrast their characteristics from tables, graphs and equations.**

Example: Graph $y = x^2 + 2$ and $y = x + 2$ on the same coordinate plane. How does the graph of $y = x^2 + 2$ differ from the graph of $y = x + 2$?

8.2.8 **Use linear functions and linear equations to represent, analyze and solve problems.**

Example: Corey is hosting a birthday party for a friend at a park shelter. The cost to rent the shelter is $50 plus $2 per person. Write a function to represent the situation. Find the total cost if 35 people attend.

Standard 3

Geometry and Measurement

CORE STANDARD

Geometry and Measurement

Constructions and Properties of Shapes

Perform basic compass and straightedge constructions: constructions of angle and segment bisectors, copies of segments and angles, and perpendicular segments. Justify the constructions. Identify properties of geometric objects.

[Standard Indicators: 8.3.1, 8.3.2]

Pythagorean Theorem

Use the Pythagorean Theorem and its converse to calculate lengths of line segments.

[Standard Indicator: 8.3.3]

Rates

Solve simple problems involving rates and derived measurements like speed and density. Express these measurements in a given unit in terms of other units within the same measurement system.

[Standard Indicators: 8.3.4, 8.3.8]

Solids

Find and use the surface areas and volumes of cones, spheres and pyramids. Use scale factors to find areas and volumes of similar figures.

[Standard Indicators: 8.3.5, 8.3.6]

8.3.1 Perform basic compass and straightedge constructions: angle and segment bisectors, copies of segments and angles, and perpendicular segments. Describe and justify the constructions.

Example: Explain the procedures used to construct the three angle bisectors of a triangle.

8.3.2 Identify, define and describe properties of three-dimensional geometric objects, describe how two or more figures intersect in a plane or in space, and visualize or describe the cross section of a solid.

Example: Find two lines in your classroom that are not parallel and yet do not meet.

8.3.3 Explain why the Pythagorean Theorem is valid using a variety of methods and use the Pythagorean Theorem and its converse to calculate lengths of line segments.

Example: A square piece of glass 7 feet on each side must be delivered through a doorway. Can the glass fit through the doorway that is 3 feet wide and 6.5 feet tall? Explain your thinking. Make a scale drawing on grid paper to solve the problem.

8.3.4 Solve simple problems involving rates and other derived measurements — including problems involving speed, uniform speed, average speed and density — by applying the concept of proportionality to measurement in different contexts. Express measurements in a given unit or in terms of other units of the same type.

Example: A car travels at 60 mph for 20 minutes and then at 48 miles an hour for 10 minutes. What is the average speed in miles per hour for this trip? Explain your answer.

8.3.5 Use scale factors to find the area and volume of similar figures.

Example: Calculate the volume and surface area of cubes with sides 1 cm, 2 cm, 3 cm, etc. Make a table of your results and describe any patterns in the table.

8.3.6 Find and use the surface area and volume of cones, spheres and pyramids.

Example: A prism has as its base a right triangle with the shorter sides of length 6 and 8 feet. Its height is 14 feet and the non-triangular faces are rectangles. Find the surface area of the prism.

8.3.7 Estimate and compute the area of irregular two-dimensional shapes and the volume of irregular three-dimensional objects by breaking them down into more basic geometric objects.

Example: Find the volume of a doghouse that has a rectangular space that is 3 ft long by 2 ft wide by 5 ft high and has a triangular roof that is 1.5 ft higher than the walls of the house.

8.3.8 Solve problems involving conversions within the same measurement system. Estimate the measure of an object in one system given the measure of that object in another system and the approximate conversion factor.

Example: The area of a hall is 40 square yards. What is the area in square feet?

Standard 4

Data Analysis and Probability

CORE STANDARD

Data Analysis and Probability

Evaluating Claims, Selecting Samples and Analyzing Bias

Identify claims based on statistical data and in simple cases evaluate the reasonableness of the claims. Identify different methods of selecting samples. Analyze the strengths and weaknesses of each method and the possible bias in samples or displays.

[Standard Indicators: 8.4.1, 8.4.2]

Analyzing Data

Use mean, median, mode, upper and lower quartiles, and range of data to compare data sets. Organize and display data to analyze central tendencies of data. Investigate effects of change in data values on the measures of the central tendency of the set of data. Display data in scatter plots and informally find lines of best fit.

[Standard Indicators: 8.4.3, 8.4.5]

Simple Experiments

Compute probabilities of events from simple experiments with equally probable outcomes.

[Standard Indicator: 8.4.7]

8.4.1 **Identify claims based on statistical data and in simple cases evaluate the reasonableness of the claims. Design a study to investigate the claim.**

Example: A study shows that teenagers who use a certain brand of toothpaste have fewer cavities than those using other brands. Describe how you can test this claim in your school.

8.4.2 **Identify different methods of selecting samples. Analyze the strengths and weaknesses of each method and the possible bias in samples or displays.**

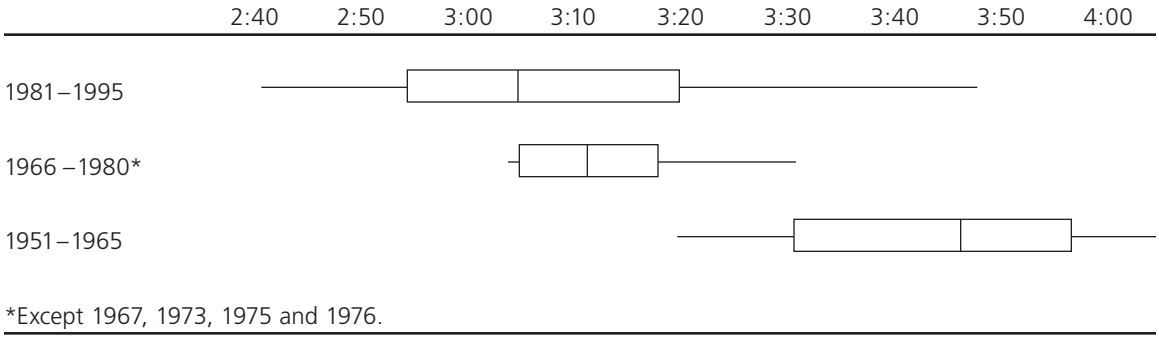
Example: Describe possible bias in the following survey: A local television station has a daily call-in poll. Viewers of the morning and noon newscasts are asked to call one telephone number to answer "yes" and a different telephone number to answer "no." The results are reported on the five-o'clock newscast.

8.4.3 **Use mean, median, mode, upper and lower quartiles, and range to compare data sets. Organize and display data to highlight important features such as the range and how the data is spread around a central value. Investigate what happens to the display when some of the data values are changed.**

Example: Arrange a set of test scores in increasing order and find the lowest and highest scores, the median, and the upper and lower quartiles.

8.4.4 **Analyze, interpret and display data in box-and-whisker plots.**

Example: The box-and-whisker plots below show winning times (hours: minutes) for the Indianapolis 500 race in selected years:



*Except 1967, 1973, 1975 and 1976.

In the years from 1951 to 1965, the slowest time was 3 hours 57 minutes. Explain how the slowest time changed in the years 1951 to 1995. How did winning times change during that period? How did the median times change in the same period?

8.4.5 **Display two-variable data in scatter plots and describe how the data points are distributed. If the pattern appears to be linear, draw a line that appears to best fit the data and write the equation of that line.**

Example: Survey some of the students at each grade level in your school to ask them how much time they spend on homework. Plot the grade level and time of each student as a point (grade, time) on a scatter diagram. Describe and justify any relationship between grade and time spent on homework.

8.4.6 **Describe and apply the addition rule for probabilities for simple events that are mutually exclusive and for simple events that are not.**

Example: Amy and Bill were each asked to roll two six-sided dice and add the numbers shown. Amy wins if the sum is odd. Bill wins if the sum is six, seven or eight. Who is more likely to win? Show your work.

8.4.7 **Compute probabilities of events from simple experiments with equally probable outcomes. Use methods like organized lists, tree diagrams and area models.**

Example: A six-sided die is rolled, and a spinner with equal sections A, C, D and E is spun. Find the probability of rolling a 3 and spinning a vowel.

PROCESS STANDARDS

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The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

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- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Round numbers to a specified place value.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurements in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning and problem-solving skills.
    - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
    - The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

American Diploma Project (2004). *Ready or Not: Creating a High School Diploma that Counts.* Washington, D.C.: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston VA: author.

NOTES

Standard 1

Relations and Functions

CORE STANDARD
Relations and Functions

Determine whether a relation is a function or not a function. Identify the domain and range of a given relation. Translate among tables, graphs, words and equations.

[Standard Indicators: A1.1.1, A1.1.2]

A1.1.1 **Determine whether a relation represented by a table, graph, words or equation is a function or not a function. Translate among tables, graphs, words and equations.**

Example: For a square of side x, the area y is given by $y = x^2$. Is y a function of x? Is x a function of y? Answer the same questions for $y = x^2$ if you are told that this equation holds for negative as well as positive values of x.

A1.1.2 **Identify the domain and range of relations represented by tables, graphs, words and equations.**

Example: What is the largest domain for x when $y = x^2$? What is the range of y in this case?

Standard 2

Linear Functions, Equations and Inequalities

CORE STANDARD
Linear Equations and Inequalities

Graph linear functions and determine their slopes and *x*- and *y*-intercepts from their graphs and equations. Write a linear function in slope-intercept form. Determine the equation of a line given sufficient information.

[Standard Indicators: A1.2.2, A1.2.3, A1.2.4]

A1.2.1 **Translate among various representations of linear functions like tables, graphs, words and equations.**

Example: Use a spreadsheet to create a table of values for the function $y = -\frac{1}{2}x + 5$. Graph the function.

A1.2.2 **Graph linear equations and show they have constant rates of change.**

Example: Kathy borrowed \$80 from her mother and plans to pay her mother \$10 per week until the debt is paid. The equation for the amount of money Kathy owes her mother is $y = 80 - 10x$, where x is the number of weeks after the loan. Graph the equation. What does the slope of the graph represent?

A1.2.3 **Determine the slope, x-intercept and y-intercept of a line given its graph, its equation or two points on the line. Then determine the equation of a line given sufficient information.**

Example: Find the slope and y-intercept of the line $4x + 6y = 12$.

A1.2.4 **Write, interpret and translate among equivalent forms of equations for linear functions (i.e., slope-intercept, point-slope and standard). Recognize that equivalent forms reveal more or less information about a given situation.**

Example: Write the equation of the line $4x + 6y = 12$ in slope-intercept form. What is the slope of this line? Explain your answer.

A1.2.5 **Solve problems that can be modeled using linear equations and inequalities, interpret the solutions and determine whether the solutions are reasonable.**

Example: As your family is traveling along an interstate, you note the distance traveled every five minutes. The distance is approximately the same. You graph the distance traveled as a function of time and assume that what was found for five-minute time intervals holds for all time intervals up to two hours. Draw a linear graph representing this trip. Predict the time of a journey of 50 miles. What does the slope of the graph represent?

A1.2.6 **Graph a linear inequality in two variables.**

Example: Draw the graph of the inequality $6x + 8y \geq 24$ on a coordinate plane.

Standard 3

Pairs of Linear Equations and Inequalities

CORE STANDARD

Pairs of Linear Equations in Two Variables

Solve pairs of linear equations in two variables by graphing, substitution or elimination. Solve problems that can be modeled using pairs of linear equations in two variables.

[Standard Indicators: A1.3.1, A1.3.3]

CORE STANDARD

Pairs of Linear Inequalities in Two Variables

Graph the solution for pairs of linear inequalities in two variables.

[Standard Indicator: A1.3.2]

A1.3.1 **Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing, substitution or elimination.**

Example: Solve the system of equations $2y + x = 10$ and $x = y + 3$. Graph the two lines and label the point of intersection.

A1.3.2 **Graph with and without technology the solution set for a pair of linear inequalities in two variables. Use the graph to find the solution set.**

Example: Graph the inequalities $y \leq 4$ and $x + y \leq 5$. Shade the region where both inequalities are true.

A1.3.3 **Solve problems that can be modeled using pairs of linear equations in two variables, interpret the solutions and determine whether the solutions are reasonable.**

Example: The income a company makes from a certain product can be represented by the equation $y = 10.5x$, and the expenses for that product can be represented by the equation $y = 5.25x + 10{,}500$, where x is the number of units of the product sold and y is the number of dollars. How many units of the product must be sold for the company to reach the break-even point?

Standard 4

Polynomials

CORE STANDARD
Rational Exponents

Understand and use the laws of exponents for variables with exponents. Multiply, divide and find powers of variables with exponents.

[Standard Indicator: A1.4.1]

CORE STANDARD
Polynomials

Multiply polynomials, factor polynomials and divide a polynomial by a monomial.

[Standard Indicators: A1.4.2, A1.4.3]

A1.4.1 Use the laws of exponents for variables with exponents. Multiply, divide and find powers of variables with exponents.

Example: Simplify $a^2b^6\ (a^3)$, $(n+2)\ (n-2)$, and $(n+2)^2$.

A1.4.2 Add, subtract and multiply polynomials and divide polynomials by monomials.

Example: Subtract $(4x^2-7x+2)-(x^2+4x-5)$, multiply $(n+2)(4n-5)$ and divide $4x^3y^2+8xy^4-6x^2y^5$ by $2xy^2$.

A1.4.3 Factor common terms from polynomials and factor quadratic expressions.

Example: Factor $4ax+3ay+4bx+3by$, $2x^2-7x+3$ and $9x^2-4$.

Standard 5

Quadratic Equations and Functions

CORE STANDARD

Quadratic Equations and Functions

Solve quadratic equations by graphing, factoring and using the quadratic formula. Graph quadratic functions and understand the relationship between its zeros and the *x*-intercepts of its graph. Solve problems that can be modeled using quadratic equations.

[Standard Indicators: A1.5.1, A1.5.2, A1.5.3, A1.5.4]

A1.5.1 **Graph quadratic functions.**

Example: Draw the graph of $y = x^2 - 3x + 2$. Using a graphing calculator or a spreadsheet to generate a data set, display the graph to check your work.

A1.5.2 **Solve quadratic equations in the real number system with real number solutions by factoring, by completing the square and by using the quadratic formula.**

Example: Solve the equation $x^2 - x + 2 = 0$ in three ways: by factoring the polynomial, by the quadratic formula and by completing the square. Derive the general quadratic formula by applying the method of completing the square to $ax^2 + bx + c = 0$.

A1.5.3 **Solve problems that can be modeled using quadratic equations, interpret the solutions and determine whether the solutions are reasonable.**

Example: A ball falls so that its distance above the ground can be modeled by the equation $s = 100 - 16t^2$, where s is the distance above the ground in feet and t is the time in seconds. According to this model, at what time does the ball hit the ground?

A1.5.4 **Analyze and describe the relationships among the solutions of a quadratic equation, the zeros of a quadratic function, the *x*-intercepts of the graph of a quadratic function and the factors of a quadratic expression.**

Example: A graphing calculator can be used to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth. Justify using the x-intercepts of $y = 3x^2 - 5x - 1$ as the solutions of the $3x^2 - 5x - 1 = 0$.

A1.5.5 **Sketch and interpret linear and non-linear graphs representing given situations and identify independent and dependent variables.**

Example: The height (h) above water of a diver t seconds after she steps off a 100-feet-high platform is given by the formula $h = 100 - 16t$. Graph the function.

Standard 6

Rational and Radical Expressions and Equations

A1.6.1 Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions with linear and quadratic denominators, including denominators with negative exponents.

Example: Simplify $\frac{x^2-4}{x^5} \div \frac{x^3-8}{x^8}$.

A1.6.2 Solve equations involving rational expressions.

Example: Solve $\frac{x+5}{4} = \frac{3x+5}{7}$ and $\frac{8}{x} + \frac{28}{x^2-4} = \frac{7}{x-2}$.

A1.6.3 Simplify radical expressions involving square roots.

Example: Assuming that x and y represent non-negative real numbers, simplify $\sqrt{18xy^2}$.

A1.6.4 Solve equations that contain radical expressions on only one side of the equation and identify extraneous roots when they occur.

Example: Solve the equation $\sqrt{x+6} = x$.

Standard 7

Data Analysis

A1.7.1 Organize and display data using appropriate methods to detect patterns and departures from patterns. Summarize the data using measures of center (i.e., mean, median) and spread (i.e., range, percentiles, variance, standard deviation). Compare data sets using graphs and summary statistics.

Example: Design and conduct a survey about the number of electronic games owned by girls and boys in your school. Organize and display the results of your survey in an appropriate graph. Describe the technique you used to get a random sample. Find the mean, median and mode of your survey data. Which of these gives a useful summary of the data?

A1.7.2 Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest, and make inferences from sample results.

Example: To determine what type of videos its customers liked, Drake Video surveyed every tenth person to walk in the store. Describe the sampling method used by Drake Video. Is it an unbiased sampling? Explain your answer.

A1.7.3 Evaluate reports based on data published in the media by considering the source of the data, the design of the study, the way the data are analyzed and displayed and whether the report confuses correlation with causation.

Example: Find an example of a graph in a newspaper or magazine that could be considered misleading. Explain why the graph could be misleading.

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Standard 1

Functions

A2.1.1 Find the zeros, domain and range of a function.

Example: $f(x) = x^3 - 3x^2 - x + 3$. What is the domain and range of this function?

A2.1.2 Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs.

Example: Given the function f below, find the indicated value, if possible.

- where f is represented by the set of ordered pairs {(3, 5), (2, -3), (1, 7), (0, 2)}, find the value of $f(1)$.
- where $f(x) = \sqrt{4 - x}$, find the value of $f(2)$ and $f(8)$.

A2.1.3 Recognize and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph and the factors of a polynomial expression.

Example: Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

Standard 2

Linear and Absolute Value Equations, Inequalities and Functions

CORE STANDARD

Linear and Absolute Value Equations and Inequalities

Solve systems of linear equations and inequalities in three variables by substitution and elimination. Solve problems that can be modeled using systems of linear equations. Solve equations and inequalities involving absolute value.

[Standard Indicators: A2.2.1, A2.2.2, A2.2.4]

A2

A2.2.1 Solve systems of linear equations and inequalities in three variables by substitution and elimination.

Example: Solve the system of equations: $x - 2y + 3z = 5, x + 3z = 11$ and $5y - 6z = 9$.

A2.2.2 Solve problems that can be modeled using systems of linear equations in three variables, interpret the solutions and determine whether the solutions are reasonable.

Example: Each week you can work no more than 20 total hours between the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

A2.2.3 **Graph piecewise-defined functions.**

Example: Graph the function $f(x) = \begin{cases} x+2 \text{ if } x \le 0 \\ 3x-1 \text{ if } x > 0 \end{cases}$.

A2.2.4 **Solve equations and inequalities involving the absolute value of a linear function.**

Example: Solve the inequality $|x-5| \ge 8$ and graph the solution.

Standard 3

Quadratic Equations and Functions

CORE STANDARD
Complex Numbers

Add, subtract, multiply and divide complex numbers.

[Standard Indicator: A2.3.1]

CORE STANDARD
Quadratic Equations and Functions

Solving Quadratic Equations
Solve quadratic equations in the complex number system. Solve problems that can be modeled using quadratic equations and functions.

[Standard Indicators: A2.3.2, A2.3.5]

Graphing Quadratic Functions
Graph quadratic functions. Determine how the graph of a parabola changes if *a*, *b* and *c* are changed in the equation $y = a(x-b)^2 + c$. Find an equation for a parabola given sufficient information.

[Standard Indicators: A2.3.3, A2.3.4]

A2.3.1 **Define, add, subtract, multiply and divide complex numbers. Represent complex numbers and the addition, subtraction and absolute value of complex numbers in the complex plane.**

Example: Let $z = 7-4i$ and $w = 10+6i$. Graph z, w and $z+w$. Prove that the numbers $0, z, w$ and $z+w$ are the vertices of a parallelogram on the complex plane.

A2.3.2 **Solve quadratic equations in the complex number system.**

Example: Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

A2.3.3 **Analyze, describe and sketch graphs of quadratic functions and include the lines of symmetry.**

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

A2.3.4 **Determine how the graph of a parabola changes if a, b and c change in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola when given sufficient information.**

Example: Write the equation of the parabola with vertex $(3, 6)$ and y-intercept 2 in vertex form.

A2.3.5 **Solve problems that can be modeled using quadratic equations and functions, interpret the solutions, and determine whether the solutions are reasonable.**

Examples:

- Write the equation of the parabola with vertex $(3, 6)$ and y-intercept 2 in vertex form.
- Describe similarities and differences in the graphs of $y = 2x^2$ and $y = 2(x - 1)^2 + 3$ without first graphing the equations.

Standard 4

Polynomial Expressions, Equations and Functions

CORE STANDARD

Polynomial Equations and Functions

Solving Polynomial Equations

Solve polynomial equations by factoring. Solve problems that can be modeled using polynomial equations.

[Standard Indicators: A2.4.4, A2.4.6]

Writing Polynomial Equations

Perform arithmetic operations, including long division, on polynomials. Find a polynomial when given its roots and use the relationships among solutions of an equation, zeros of a function, *x*-intercepts of a graph and factors of a polynomial expression to solve problems.

[Standard Indicators: A2.4.3, A2.4.7]

A2

A2.4.1 **Analyze, describe and sketch graphs of polynomial functions by examining intercepts, zeros, domain and range, and end behavior.**

Example: Determine by inspection the end behavior of the graph of the function. $f(x) = -2x^3 + x^2 + 4x - 5$.

A2.4.2 **Use the binomial theorem to expand binomial expressions raised to positive integer powers.**

Example: Expand $(x + 2)^4$.

A2.4.3 **Perform arithmetic operations, including long division and division with remainders, on polynomials by others of equal or lower degree.**

Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

A2.4.4 **Factor polynomials completely and solve polynomial equations by factoring.**

Example: Solve $x^3 + 27 = 0$ by factoring.

A2.4.5 **Use graphing technology to find approximate solutions for polynomial equations.**

Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

A2.4.6 **Solve problems that can be represented or modeled using polynomial equations, interpret the solutions and determine whether the solutions are reasonable.**

Example: You want to make an open-top box with a volume of 500 cubic inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Then use your results to give a formula for the volume of the box.

A2.4.7 **Find a polynomial function of lowest degree with real coefficients when given its roots. Solve problems by using the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph and factors of a polynomial expression.**

Example: Write an equation that has solutions $x = 2, x = 5i,$ and $x = -5i$.

Standard 5

Rational and Radical Expressions, Equations and Functions

CORE STANDARD
Rational Functions

Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator. Solve problems that can be modeled using equations involving rational functions.

[Standard Indicator: A2.5.2]

A2

A2.5.1 **Analyze, describe and sketch graphs of rational functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.**

Example: Find the equations of the horizontal and vertical asymptotes of the function $f(x) = \dfrac{x+1}{x+5}$.

A2.5.2 **Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator.**

Example: Simplify $\dfrac{x^2-4}{x^5} \div \dfrac{x^3-8}{x^8}$.

A2.5.3 Understand the properties of rational exponents and use the properties to simplify, multiply, divide and find powers of expressions containing negative and fractional exponents. Relate expressions containing rational exponents to the corresponding radical expressions.

Example: Write the expression $\left(x^{\frac{1}{2}}\ y^{\frac{-2}{3}}\right)^6$ in simplest form. Assume all variables are positive.

A2.5.4 Analyze, describe and sketch graphs of square root and cube root functions by examining intercepts, zeros, domain and range, and end behavior.

Example: Graph the function $y = \sqrt{x} + 7$ and find the domain and range.

A2.5.5 Solve equations that contain radical expressions and identify extraneous roots when they occur.

Example: Solve the equation $x = \sqrt{x} + 2$.

A2.5.6 Solve problems that can be modeled using equations involving rational and radical functions, including problems of direct and inverse variation. Interpret the solutions and determine whether the solutions are reasonable.

Example: Two students working independently can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together at the same rate as when doing the job alone?

Standard 6

Exponential and Logarithmic Functions

CORE STANDARD

Exponential and Logarithmic Equations

Use laws of exponents to derive laws of logarithms. Use laws of logarithms to solve problems. Solve exponential and logarithmic equations. Solve problems that can be modeled using equations involving exponents and logarithms.

[Standard Indicators: A2.6.2, A2.6.3, A2.6.4]

A2

A2.6.1 Analyze, describe and sketch graphs of exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

A2.6.2 Know that the inverse of an exponential function is a logarithm. Use laws of exponents to derive laws of logarithms. Use the inverse relationship between exponential functions and logarithms and the laws of logarithms to solve problems.

Example: If you know that $\log(2) = a$ and $\log(3) = b$, find $\log(36)$ in terms of a and b.

A2.6.3 Solve exponential and logarithmic equations.

Example: Solve the equation $\log_2 x = 5$.

A2.6.4 **Solve problems that can be modeled using exponential and logarithmic equations, interpret the solutions, and determine whether the solutions are reasonable. Use technology as appropriate.**

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million and 400 million. What do you notice about these time periods?

Standard 7

Sequences and Series

CORE STANDARD

Sequences and Series

Find specific terms of arithmetic and geometric sequences. Find partial sums of arithmetic and geometric series. Solve problems that can be modeled using arithmetic and geometric series.

[Standard Indicators: A2.7.1, A2.7.3, A2.7.4]

A2.7.1 **Write the recursive formula for arithmetic and geometric sequences and find specific terms of arithmetic and geometric sequences.**

Example: Find the tenth term of the arithmetic sequence 3, 7, 11, 15… .

A2.7.2 **Write the formula for the general term for arithmetic and geometric sequences and make connections to linear and exponential functions.**

Example: Write the formula for the general term of the geometric sequence 2, 6, 18, 54, 162… .

A2.7.3 **Find partial sums of arithmetic and geometric series.**

Example: In the last example (A2.7.2), find the sum of the first 10 terms.

A2.7.4 **Solve problems involving applications that can be modeled using sequences and finite arithmetic and geometric series. Interpret the solutions and determine whether the solutions are reasonable using spreadsheets as appropriate.**

Example: A restaurant has square tables that seat four people. When two tables are placed together, six people can be seated. If 20 square tables are placed together to form one long table, how many people can be seated?

Standard 8

Data Analysis and Probability

CORE STANDARD

Combinatorics and Probability

Use permutations, combinations and other counting methods to determine the number of ways that events can occur. Calculate the probability of compound events and analyze probabilities to interpret odds and risks of events.

[Standard Indicators: A2.8.2, A2.8.4]

A2.8.1 **Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.**

Example: Use technology to simulate throwing two dice 500 times. Use the results to estimate the probability of rolling a 7 and then use the diagram of the sample space to find the theoretical probability.

A2.8.2 **Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risks of events.**

Example: When a die is rolled three times, what is the probability of obtaining a 6, followed by any even number, followed by a 4?

A2.8.3 **Know and apply the characteristics of the normal distribution.**

- **Identify settings in which the normal distribution may be useful.**
- **Determine whether a set of data appears to be uniform, skewed or normally distributed.**
- **Use the empirical rule to find probabilities that an event will occur in a specific interval that can be described in terms of one, two or three standard deviations from the mean.**

Example: Math SAT scores are normally distributed with mean 500 and standard deviation 100. What is the probability that a randomly selected student's SAT score is greater than 600?

A2.8.4 **Use permutations, combinations and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.**

Example: There are five students who work in a bookshop. If the bookshop needs three people to operate, how many days straight could the bookstore operate without the same group of students working twice?

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The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
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- Use the language of mathematics to express mathematical ideas precisely.

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- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

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- Create and use representations to organize, record and communicate mathematical ideas.
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- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
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- Decide when estimation is an appropriate strategy for solving a problem.
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- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
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References

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NOTES

Standard 1

Points, Lines, Angles and Planes

CORE STANDARD
Coordinate Geometry

Find slopes, lengths and midpoints of line segments using coordinate geometry. Use these measures to show whether shapes are similar or congruent and whether line segments are parallel or perpendicular. Find the equation of a circle in the coordinate plane.

[Standard Indicators: G.1.1, G.1.4, G.1.6, G.3.5]

CORE STANDARD
Angles and Lines

Understand the relationship between special angles created by parallel lines and transversals.

[Standard Indicator: G.1.3]

G.1.1 **Find the length of line segments in one- or two-dimensional coordinate systems, the slopes of line segments in two-dimensional coordinate systems, and the point that is a given fractional distance from one end of the segment to another.**

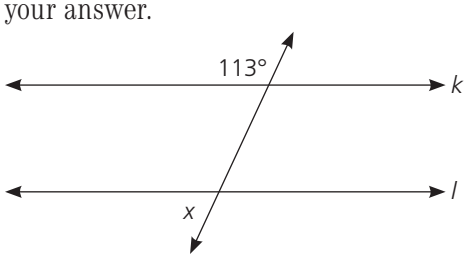
Example: Find the length of the line segment joining $A(3, 8)$ and $B(9, 0)$. Find the midpoint of this segment and the point that is one-third of the way from A to B.

G.1.2 **Construct congruent segments and angles, angle bisectors, perpendicular bisectors, and parallel and perpendicular lines by using appropriate geometric construction tools. Explain and justify the process used.**

Example: Construct the perpendicular bisector of a given line segment, justifying each step of the process.

G.1.3 **Recognize, use and justify the relationships between special pairs of angles formed by parallel lines and transversals.**

Example: In the diagram, the lines k and l are parallel. What is the measure of angle x? Explain your answer.



G.1.4 **Identify and apply properties of and theorems about parallel and perpendicular lines, write equations of parallel and perpendicular lines, and develop simple geometric proofs involving parallel and perpendicular lines.**

Example: Find an equation of a line perpendicular to $y = 4x - 2$ that contains the point (4, 1).

G.1.5 **Identify, justify and apply properties of planes.**

Example: Describe the intersection of plane R with parallel planes S and T.

G.1.6 **Represent geometric objects and figures algebraically using coordinates, use algebra to solve geometric problems, and develop simple coordinate proofs involving geometric objects in the coordinate plane.**

Example: Draw a triangle with vertices at (1, 3), (2, 5) and (6, 1). Draw another triangle with vertices (-3, -1), (-2, 1) and (2, -3). Are these triangles the same shape and size? Justify your answer.

G.1.7 **Describe the intersection of two or more geometric figures in the plane.**

Example: What is the maximum number of times two circles of the same size can intersect? Three circles? Six circles? Explain your reasoning.

Standard 2

Polygons

CORE STANDARD
Polygons

Find the sum of the measures of the interior and exterior angles of convex polygons. Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons. Understand how limiting cases of such formulas leads to expressions for the circumference and the area of a circle.

[Standard Indicators: G.2.1, G.2.5]

CORE STANDARD
Congruence and Similarity

Develop simple geometric proofs involving congruent and similar polygons. Solve problems involving congruent and similar polygons and solids.

[Standard Indicators: G.2.3, G.2.7, G.2.12, G.4.2]

CORE STANDARD
Transformations

Predict and describe the results of translations, reflections and rotations. Describe a motion or series of motions that will show that two shapes are congruent.

[Standard Indicator: G.2.4]

CORE STANDARD
Geometric Proof and Reasoning

Understand the differences among supporting evidence, counterexamples and actual proofs. Be able to develop simple geometric proofs, providing reasons for each statement, involving the following topics:

- parallel lines and transversals;
- congruent and similar polygons, particularly triangles;
- circles; and
- geometric objects in the coordinate plane.

[Standard Indicators: G.1.4, G.2.7, G.2.14, G.3.6, G.5.3, G.5.4]

CORE STANDARD
Triangles

Prove the Pythagorean Theorem and its converse and use them to solve problems. Develop simple geometric proofs involving triangles. Define trigonometric functions in terms of angles of right triangles and use them to solve problems.

[Standard Indicators: G.2.14, G.2.16, G.2.17, G.2.19, G.2.21]

G

General

G.2.1 **Justifying the method used, find and use the sum of the measures of interior and exterior angles of convex polygons.**

Example: Calculate the measure of one interior angle of a regular octagon. Explain your method.

G.2.2 **Identify types of symmetry (i.e., line, point, rotational, self-congruences) of polygons.**

Example: $ABCD$ is a rhombus. Identify and describe its reflection and rotation symmetry.

G.2.3 Solve problems involving congruent and similar polygons.

Example: In the figures below, Figure A ≅ Figure B. Justify this statement: $\angle x \cong \angle y$.

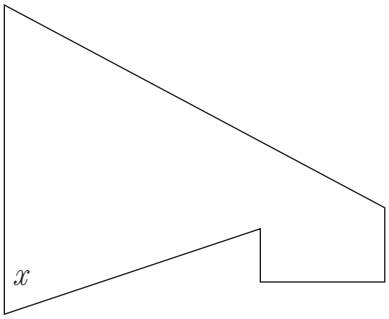


Figure A

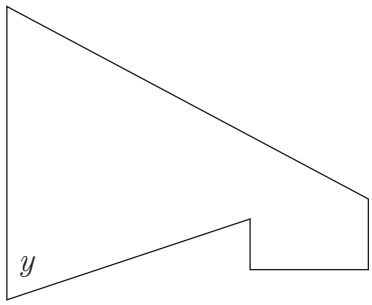


Figure B

G.2.4 Predict and describe the results of translations, reflections and rotations on polygons. Describe a motion or series of motions that will show that two shapes are congruent.

Example: Use a drawing program to create quadrilaterals and regular hexagons, octagons and pentagons. Under the drawings, describe which of the polygons tessellate. From your drawings, can you find a set of polygons in which all within the set tessellate? Show how you determined this.

G.2.5 Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons. Understand how limiting cases of such formulas lead to expressions for the circumference and the area of a circle.

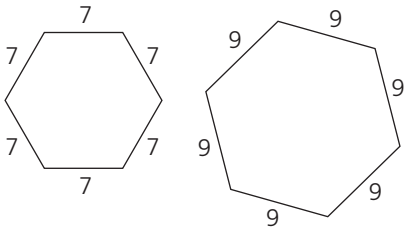
Example: Use trigonometric functions to find the perimeter and the area of a regular 12-gon that has been inscribed in a circle with radius r.

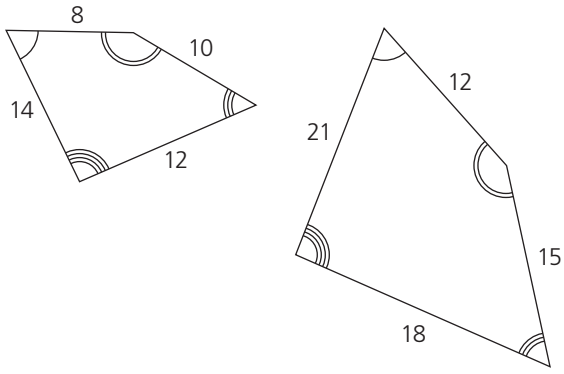
G.2.6 Recognize and use coordinate geometry to verify properties of polygons such as regularity, congruence and similarity.

Example: Is the polygon formed by connecting the points (2, 1), (6, 2), (5, 6) and (1, 5) a square?

G.2.7 Develop simple geometric proofs involving congruent and similar polygons and provide reasons for each statement.

Example: Prove that the following pairs of polygons are similar.





Quadrilaterals

G.2.8 Describe, classify and recognize relationships among the quadrilaterals, such as squares, rectangles, rhombuses, parallelograms, trapezoids and kites.

Example: Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid and kite. Judge which of the quadrilaterals has perpendicular diagonals and draw those diagonals in the figures. Give a convincing argument that your judgment is correct.

G.2.9 **Prove and apply theorems about parallelograms and trapezoids (including isosceles trapezoids) involving their angles, sides and diagonals. Prove that the given quadrilaterals are parallelograms, rhombuses, rectangles, squares or trapezoids (as appropriate).**

Example: Prove that both pairs of opposite sides of a parallelogram are congruent.

Triangles

G.2.10 **Define, identify, construct and solve problems involving perpendicular bisectors, angle bisectors, medians and altitudes in triangles.**

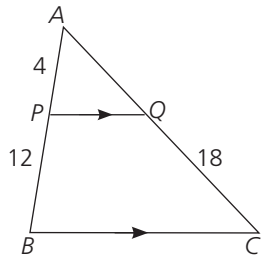
Example: Draw several triangles. Construct their angle bisectors. What do you notice?

G.2.11 **Construct triangles congruent to given triangles. Explain and justify the process used.**

Example: Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

G.2.12 **Use theorems to show if two triangles are congruent (i.e., SSS, SAS, ASA) or similar (i.e., AA, SAS, SSS).**

Example: In the example below, prove that ΔABC and ΔAPQ are similar and use the similar triangles to compute the length of $\overline{AQ}$.



G.2.13 **Prove and apply the triangle inequality theorem.**

Example: Can 7, 15 and 5 be the sides of a triangle? Explain how you know your answer is accurate.

G.2.14 **Develop simple geometric proofs involving triangles and provide reasons for each statement of the proof.**

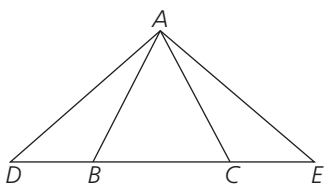
Example: Prove that:

- The sum of the angles in a triangle is 180°.
- The line joining the midpoint of two sides of a triangle is parallel to, and half the length of, the third side.
- The perpendicular bisectors of the sides of a triangle meet at a point that is the center of the circle and that contains the vertices of the triangle.

Isosceles Triangles

G.2.15 **Prove and apply the isosceles triangle theorem and its converse.**

Example: Given isosceles ΔABC and $DB \cong EC$, prove $\Delta DBA \cong \Delta ECA$.



Right Triangles

G.2.16 **Prove the Pythagorean Theorem and its converse and use them to solve problems, including problems involving the length of a segment in the coordinate plane.**

Example: Triangle DEF has vertices $D(2, 4)$, $E(0, 2)$, and $F(3, -1)$. Determine whether ΔDEF is a right triangle.

G.2.17 **Prove and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle.**

Example: In ΔABC with right angle at C, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.

G.2.18 **Use special right triangles (e.g., 30° - 60° and 45° - 45°) to solve problems.**

Example: An isosceles right triangle has one short side of 6 cm. Find the lengths of the other two sides.

G.2.19 **Define and use the trigonometric functions sine, cosine and tangent in terms of angles of right triangles.**

Example: In ΔABC, $\tan A = \frac{1}{5}$. Find $\sin A$ and $\cos A$.

G.2.20 **Deduce and apply the area formula $A = \frac{1}{2}ab \sin C$, where a and b are the lengths of two sides of a triangle and C is the measure of the included angle formed by the two sides.**

Example: Find the area of an equilateral triangle with sides five units long.

G.2.21 **Solve problems that can be modeled using right triangles, including problems that can be modeled using trigonometric functions. Interpret the solutions and determine whether the solutions are reasonable. Use technology as appropriate.**

Example: The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill. What is the force on a 3,000-pound car on a hill with a 1 in 5 grade? (A grade of 1 in 5 means that the hill rises one unit for every five horizontal units.)

Standard 3

Circles

G

CORE STANDARD
Circles

Define, deduce and use formulas for and prove theorems for:

- radius, diameter, arc, chord, secant and tangent;
- measures of arcs and related angles (central, inscribed, and intersections of secants and tangents); and
- circumference, arc length and areas of circles and sectors.

Determine how the graph of a circle changes if a, b and r are changed in the equation $(x - a)^2 + (y - b)^2 = r^2$.

[Standard Indicators: G.3.2, G.3.3, G.3.4, G.3.5, G.3.6]

CORE STANDARD
Coordinate Geometry

Find slopes, lengths and midpoints of line segments using coordinate geometry. Use these measures to show whether shapes are similar or congruent and whether line segments are parallel or perpendicular. Find the equation of a circle in the coordinate plane.

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[Standard Indicators: G.1.4, G.2.7, G.2.14, G.3.6, G.5.3, G.5.4]

G.3.1 **Construct the circle that passes through three given points not on a line. Construct tangents to circles. Circumscribe and inscribe circles. Justify the process used.**

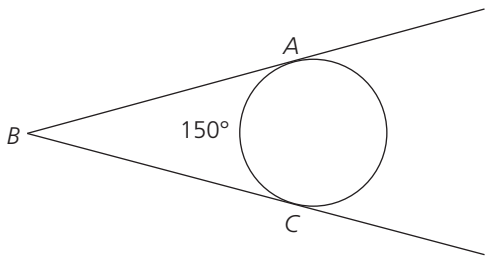
Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords.

G.3.2 **Define, deduce and use formulas for, and prove theorems for radius, diameter, chord, secant and tangent.**

Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle?

G.3.3 **Define, deduce and use formulas for, and prove theorems for measures of arcs and related angles (i.e., central, inscribed and intersections of secants and tangents).**

Example: Find the measure of $\angle ABC$ in the diagram below.



G.3.4 **Define, deduce and use formulas for, and prove theorems for measures of circumference, arc length, and areas of circles and sectors.**

Example: Use an appropriate theorem to find the sum of the interior angles of a convex n-gon.

G

G.3.5 Find the equation of a circle in the coordinate plane in terms of its center and radius and determine how the graph of a circle changes if a, b and r change in the equation $(x-a)^2+(y-b)^2=r^2$.

Example: Find the equation of the circle with radius 10 and center (6, -3).

G.3.6 Develop simple geometric proofs involving circles and provide reasons for each statement.

Example: Prove that an inscribed angle in a circle is half the measure of the central angle with the same arc.

Standard 4

Polyhedra and Other Solids

CORE STANDARD
Congruence and Similarity

Develop simple geometric proofs involving congruent and similar polygons. Provide reasons for each statement. Solve problems involving congruent and similar polygons and solids.

[Standard Indicators: G.2.3, G.2.7, G.2.12, G.4.2]

CORE STANDARD
Solids

Find and use measures of sides, volumes of solids and surface areas of solids. Relate these measures to each other using formulas.

[Standard Indicator: G.4.3]

G

G.4.1 Identify, justify and apply properties of prisms, regular pyramids, cylinders, right circular cones and spheres.

Example: Which of these properties of a cylinder is not true, and how do you know?

- The bases are congruent.
- The sections produced by the intersection of a cylinder and two parallel planes are congruent.
- The volume is the product of the area of the base and the altitude.
- The lateral area of a right circular cylinder is the product of the altitude and the base.

G.4.2 Solve problems involving congruent and similar solids.

Example: Explain how the surface area and volume of similar cylinders are related.

G.4.3 Find and use measures of sides, volumes and surface areas of prisms, regular pyramids, cylinders, right circular cones and spheres. Relate these measures to each other using formulas.

Example: A marble is dropped into a glass that is roughly a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the marble?

G.4.4 Visualize solids and surfaces in three-dimensional space when given two-dimensional representations, and create two-dimensional representations for the surfaces of three-dimensional objects.

Example: Make a net for a tetrahedron out of poster board and fold it to make the tetrahedron.

Standard 5

Geometric Reasoning and Proof

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G.5.1 Describe the structure of and relationships within an axiomatic system (e.g., undefined terms, definitions, axioms and postulates, methods of reasoning, and theorems).

Example: Do you prove axioms from theorems or theorems from axioms?

G.5.2 Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true. Illustrate its counterparts in other geometries.

Example: Describe and illustrate at least one non-Euclidean geometry postulate.

G.5.3 Understand the differences among supporting evidence, counterexamples and actual proofs.

Example: Draw and label a figure for the conjecture, "If an angle bisector of a triangle is also an altitude, then the triangle is isosceles." Support your conjecture with supporting evidence. Then write a simple proof for your conjecture.

G.5.4 Develop simple geometric proofs (i.e., direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry) using two-column, paragraphs and flow charts formats. Provide reasons for each statement in the proofs.

Example: Prove that the medians of a triangle meet at a point which is $\frac{2}{3}$ of the way from a vertex to the opposite side.

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NOTES

Standard 1

Algebra and Functions

CORE STANDARD
Relations and Functions

Determine whether a relation is a function or not a function. Identify the domain and range of a given relation. Translate among tables, graphs, words and equations.

[Standard Indicators: IM1.1.1, IM1.1.2, IM1.1.3]

CORE STANDARD
Linear Equations and Inequalities

Graphing and Writing Linear Equations
Graph linear functions and determine their slopes and *x*- and *y*-intercepts from their graphs and equations. Write a linear function in slope-intercept form. Determine the equation of a line given sufficient information.

[Standard Indicators: IM1.1.4, IM1.1.5, IM1.1.6]

Pairs of Linear Equations in Two Variables
Solve pairs of linear equations in two variables by graphing, substitution or elimination. Solve problems that can be modeled using pairs of linear equations in two variables.

[Standard Indicators: IM1.1.9, IM1.1.11]

Pairs of Linear Inequalities in Two Variables
Graph the solution for pairs of linear inequalities in two variables.

[Standard Indicator: IM1.1.10]

CORE STANDARD
Rational Exponents

Understand and use the laws of exponents for variables with exponents. Multiply, divide and find powers of variables with exponents.

[Standard Indicator: IM1.1.12]

CORE STANDARD
Quadratic Equations and Functions

Solve quadratic equations by graphing, factoring and using the quadratic formula. Graph quadratic functions and understand the relationship between its zeros and the *x*-intercepts of its graph. Solve problems that can be modeled using quadratic equations.

[Standard Indicators: IM1.1.12, IM1.1.13, IM1.1.14, IM1.1.15]

IM1.1.1 **Determine whether a relation represented by a table, graph, words or equation is a function or not a function. Translate among tables, graphs, words and equations.**

Example: For a square of side x, the area y is given by $y = x^2$. Is y a function of x? Is x a function of y? Answer the same questions for $y = x^2$ if you are told that this holds for negative as well as positive values of x.

IM1.1.2 **Identify the domain and range of relations represented by tables, graphs, words and equations.**

Example: What is the largest domain for x when $y = x^2$? What is the range of y in this case?

IM1.1.3 **Translate among various representations of linear functions like tables, graphs, words and equations.**

Example: Use a spreadsheet to create a table of values for the function $y = -\frac{1}{2}x + 5$. Graph the function.

IM1.1.4 **Graph linear equations and show they have constant rates of change.**

Example: Kathy borrowed $80 from her mother and plans to pay her mother $10 per week until the debt is paid. The equation for the amount of money Kathy owes her mother is $y = 80 - 10x$, where x is the number of weeks after the loan. Graph the equation. What does the slope of the graph represent?

IM1.1.5 **Determine the slope, x-intercept and y-intercept of a line given its graph, its equation or two points on the line. Then determine the equation of a line given sufficient information.**

Example: Find the slope and y-intercept of the line $4x + 6y = 12$.

IM1.1.6 **Write, interpret and translate among equivalent forms of equations for linear functions (i.e., slope-intercept, point-slope and standard). Recognize that equivalent forms reveal more or less information about a given situation.**

Example: Write the equation of the line $4x + 6y = 12$ in slope-intercept form. What is the slope of this line? Explain your answer.

IM1.1.7 **Solve problems that can be modeled using linear equations and inequalities, interpret the solutions and determine whether the solutions are reasonable.**

Example: As your family is traveling along an interstate, you note the distance traveled every five minutes. The distance is approximately the same. You graph the distance traveled as a function of time and assume that what was found for five-minute intervals holds for all time intervals up to two hours. Draw a linear graph representing this trip. Predict the time of a journey of 50 miles. What does the slope of the graph represent?

IM1.1.8 **Graph a linear inequality in two variables.**

Example: Draw the graph of the inequality $6x + 8y \geq 24$ on a coordinate plane.

IM1.1.9 **Understand the relationship between a solution of a pair of linear equations in two variables and the graphs of the corresponding lines. Solve pairs of linear equations in two variables by graphing, substitution or elimination.**

Example: Solve the system of equations: $2y + x = 10$ and $x = y + 3$. Graph the two lines and label the point of intersection.

IM1.1.10 **Graph with and without technology the solution set for a pair of linear inequalities in two variables. Use the graph to find the solution set.**

Example: Graph the inequalities $y \leq 4$ and $x + y \leq 5$. Shade the region where both inequalities are true.

IM1.1.11 **Solve problems that can be modeled using pairs of linear equations containing two variables, interpret the solutions and determine whether the solutions are reasonable.**

Example: The income a company makes from a certain product can be represented by the equation $y = 10.5x$, and the expenses for that product can be represented by the equation $y = 5.25x + 10{,}500$, where x is the amount of the product sold and y is the number of dollars. How many units of the product must be sold for the company to reach the break-even point?

IM1.1.12 **Graph quadratic functions.**

Example: Draw the graph of $y = x^2 - 3x + 2$. Using a graphing calculator or a spreadsheet to generate a data set, display the graph to check your work.

IM1.1.13 **Solve quadratic equations in the real number system with real number solutions by factoring, by completing the square and by using the quadratic formula.**

Example: Solve the equation $x^2 - x + 2 = 0$ in three ways: by factoring the polynomial, by using the quadratic formula and by completing the square. Derive the general quadratic formula by applying the method of completing the square to $ax^2 + bx + c = 0$.

IM1.1.14 **Solve problems that can be modeled using quadratic equations, interpret the solutions and determine whether the solutions are reasonable.**

Example: A ball falls so that its distance above the ground can be modeled by the equation $s = 100 - 16t^2$, where s is the distance above the ground in feet and t is the time in seconds. According to this model, at what time does the ball hit the ground?

IM1.1.15 **Analyze and describe the relationships among the solutions of a quadratic equation, the zeros of a quadratic function, the x-intercepts of a graph of a quadratic function and the factors of a quadratic expression.**

Example: A graphing calculator can be used to solve $3x^2 - 5x - 1 = 0$ to the nearest tenth. Justify using the x-intercepts of $y = 3x^2 - 5x - 1$ as the solutions of $3x^2 - 5x - 1 = 0$.

IM1.1.16 **Sketch and interpret linear and non-linear graphs representing given situations and identify independent and dependent variables.**

Example: The height (h) above water of a diver t seconds after she steps off a 100-foot-high platform is given by the formula $h = 100 - 16t$. Graph the function.

IM1.1.17 **Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions with linear and quadratic denominators, including denominators with negative exponents.**

Example: Simplify $\dfrac{x^2 - 4}{x^5} \div \dfrac{x^3 - 8}{x^8}$.

IM1.1.18 **Solve equations involving rational expressions.**

Example: Solve $\dfrac{x + 5}{4} = \dfrac{3x + 5}{7}$ and $\dfrac{8}{x} + \dfrac{28}{x^2 - 4} = \dfrac{7}{x - 2}$.

IM1.1.19 Simplify radical expressions involving square roots.

Example: Assuming that x and y represent non-negative real numbers, simplify $\sqrt{18xy^2}$.

IM1.1.20 Solve equations that contain radical expressions on only one side of the equation and identify extraneous roots when they occur.

Example: Solve the equation $\sqrt{x+6}=x$.

Standard 2

Geometry and Measurement

CORE STANDARD
Coordinate Geometry

Find slopes, lengths and midpoints of line segments using coordinate geometry.

[Standard Indicator: IM1.2.1]

CORE STANDARD
Polygons

Find the sum of the measures of the interior and exterior angles of convex polygons. Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons. Understand how limiting cases of such formulas leads to expressions for the circumference and the area of a circle.

[Standard Indicators: IM1.2.2, IM1.2.6]

CORE STANDARD
Congruence and Similarity

Solve problems involving congruent and similar polygons and solids.

[Standard Indicator: IM1.2.4]

CORE STANDARD
Transformations

Predict and describe the results of translations, reflections and rotations. Describe a motion or series of motions that will show that two shapes are congruent.

[Standard Indicator: IM1.2.5]

IM1.2.1 **Find the length of line segments in one- or two-dimensional coordinate systems, the slopes of line segments in two-dimensional coordinate systems, and the point that is a given fractional distance from one end of the segment to another.**

Example: Find the length of the line segment joining $A(3, 8)$ and $B(9, 0)$. Find the midpoint of this segment and the point that is one-third of the way from A to B.

IM1.2.2 **Justifying the method used, find and use the sum of the measures of interior and exterior angles of convex polygons.**

Example: Calculate the measure of one interior angle of a regular octagon. Explain your method.

IM1.2.3 **Identify types of symmetry (i.e., line, point, rotational, self-congruences) of polygons.**

Example: $ABCD$ is a rhombus. Identify and describe its reflection and rotation symmetry.

IM1.2.4 **Solve problems involving congruent and similar polygons.**

Example: In the figures below, Figure A $\cong$ Figure B. Justify this statement: $\angle x \cong \angle y$.

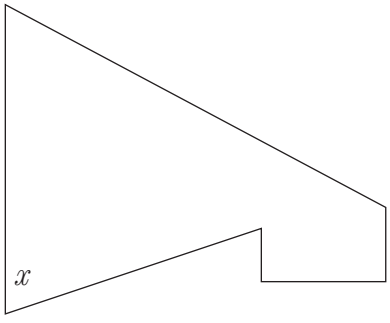


Figure A

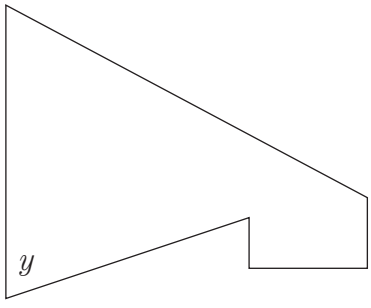


Figure B

IM1.2.5 **Predict and describe the results of translations, reflections and rotations on polygons. Describe a motion or series of motions that will show that two shapes are congruent.**

Example: Use trigonometric functions to find the perimeter and the area of a regular 12-gon that has been inscribed in a circle with radius r.

IM1.2.6 **Deduce formulas relating lengths and sides, perimeters, and areas of regular polygons. Understand how limiting cases of such formulas lead to expressions for the circumference and area of a circle.**

Example: Use the formula for the perimeter of a square to write a formula for the area of a square in terms of its perimeter.

IM1.2.7 Recognize and use coordinate geometry to verify properties of polygons such as regularity, congruence and similarity.

Example: Is the polygon formed by connecting the points (2, 1), (6, 2), (5, 6) and (1, 5) a square?

IM1.2.8 Understand the differences among supporting evidence, counterexamples and actual proofs.

Example: Draw and label a figure for the conjecture, "If an angle bisector of a triangle is also an altitude, then the triangle is isosceles." Support your conjecture with supporting evidence. Then write a simple proof for your conjecture.

IM1.2.9 Develop simple geometric proofs (i.e., direct proofs, indirect proofs, proofs by contradiction and proofs involving coordinate geometry) using two-column, paragraphs and flow charts formats. Provide reasons for each statement in the proofs.

Example: In ΔLMN, $LM = LN$. Prove that $\angle LMN \cong \angle LNM$.

Standard 3

Data Analysis and Probability

IM1.3.1 Organize and display data using appropriate methods to detect patterns and departures from patterns. Summarize the data using measures of center (i.e., mean, median) and spread (i.e., range, percentiles, variance, standard deviation). Compare data sets using graphs and summary statistics.

Example: Design and conduct a survey about the number of electronic games owned by girls and boys in your school. Organize and display the results of your survey in an appropriate graph. Describe the technique you used to get a random sample. Find the mean, median and mode of your survey data. Which of these gives a useful summary of the data?

IM1.3.2 Distinguish between random and non-random sampling methods, identify possible sources of bias in sampling, describe how such bias can be controlled and reduced, evaluate the characteristics of a good survey and well-designed experiment, design simple experiments or investigations to collect data to answer questions of interest and make inferences from sample results.

Example: To determine what type of videos its customers liked, Drake Video surveyed every tenth person to walk in the store. Describe the sampling method used by Drake Video. Is it an unbiased sampling? Explain your answer.

IM1.3.3 Evaluate reports based on data published in the media by considering the source of the data, the design of the study, the way the data are analyzed and displayed, and whether the report confuses correlation with causation.

Example: Find an example of a graph in a newspaper or magazine that could be considered misleading. Explain why the graph could be misleading.

IM1.3.4 Use the relative frequency of a specified outcome of an event to estimate the probability of the outcome and apply the law of large numbers in simple examples.

Example: Use technology to simulate throwing two dice 500 times. Use the results to estimate the probability of rolling a 7 and then use the diagram of the sample space to find the theoretical probability.

Standard 4

Discrete Mathematics

IM1.4.1 **Analyze and apply algorithms for searching (sequential, binary), for sorting (bubble sort, quick sort, bin sort) and for solving optimization problems.**

Example: Use bubble sort to put 13, 14, 12, 11, 15 in increasing order.

IM1.4.2 **Analyze and interpret relationships defined iteratively and recursively.**

Example: Use the recursive definition of the Fibonacci numbers to find the fifth term.

IM1.4.3 **Define arithmetic and geometric sequences recursively.**

Example: There are 2,500 fish in a pond. Each year the population decreases by 25 percent, but 1,000 fish are added to the pond at the end of each year. Find the population in five years. Also, find the long-term population.

IM1.4.4 **Determine the number of ways events can occur using permutations, combinations and the Fundamental Counting Principle.**

Example: You are getting dressed one morning when you realize that you have far too many choices. You have six shirts to choose from, four pairs of jeans and three pairs of shoes. Ignoring color coordination, construct a tree diagram or other pictorial representation to show how many different outfits you could assemble.

IM1.4.5 **Determine whether two propositions are logically equivalent.**

Example: Show that "If today is Sunday, then we have school tomorrow," and "It is not Sunday or we have school tomorrow," are logically equivalent.

IM1.4.6 **Distinguish between inductive and deductive reasoning. Identify inductive reasoning as central to the scientific method and deductive reasoning as characteristic of mathematics.**

Example: What type of reasoning are you using when you look for a pattern?

PROCESS STANDARDS

Indiana's Academic Standards for Mathematics describe the key content of each grade level and course, and students must develop conceptual understanding of this content. The American Diploma Project noted that, "beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and 'mathematicizing' a problem" (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
  - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
  - The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

American Diploma Project (2004). *Ready or Not: Creating a High School Diploma that Counts*. Washington, D.C.: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston VA: author.

NOTES

Standard 1

Algebra and Functions

CORE STANDARD
Linear and Absolute Value Equations and Inequalities

Solve systems of linear equations and inequalities in three variables by substitution and elimination. Solve problems that can be modeled using systems of linear equations. Solve equations and inequalities involving absolute value.

[Standard Indicators: IM2.1.4, IM2.1.5, IM2.1.7]

CORE STANDARD
Polynomials

Multiply polynomials, factor polynomials and divide a polynomial by a monomial.

[Standard Indicators: IM2.1.9, IM2.1.10]

IM2.1.1 **Use and interpret function notation, including evaluation of functions represented by tables, graphs, words, equations or a set of ordered pairs.**

Example: Given the function f below, find the indicated value, if possible.

- where f is represented by the set of ordered pairs $\{(3, 5), (2, -3), (1, 7), (0, 2)\}$, find the value of $f(1)$.
- where $f(x) = \sqrt{4-x}$, find the value of $f(2)$ and $f(8)$.

IM2.1.2 **Recognize and describe the relationships among the solutions of an equation, the zeros of a function, the x-intercepts of a graph and the factors of a polynomial expression.**

Example: Solve the equation $x^4 + x^3 - 7x^2 - x + 6 = 0$, given that $x - 2$ and $x + 3$ are factors of $x^4 + x^3 - 7x^2 - x + 6$.

IM2.1.3 **Solve systems of linear equations and inequalities in three variables by substitution and elimination.**

Example: Solve the system of equations: $x - 2y + 3z = 5, x + 3z = 11, 5y - 6z = 9$.

IM2.1.4 **Solve problems that can be modeled using systems of linear equations in three variables, interpret the solutions and determine whether the solutions are reasonable.**

Example: Each week you can work no more than 20 total hours between the local bookstore and the drugstore. You prefer the bookstore and want to work at least 10 more hours there than at the drugstore. Draw a graph to show the possible combinations of hours that you could work.

IM2.1.5 **Graph piecewise-defined functions.**

Example: Graph the function $f(x) = \begin{cases} x + 2 \text{ if } x \le 0 \\ 3x - 1 \text{ if } x > 0 \end{cases}$.

IM2

IM2.1.6 Solve equations and inequalities involving the absolute value of a linear function.

Example: Solve the inequality $|x-5| \geq 8$ and graph the solution.

IM2.1.7 Use the laws of exponents for variables with exponents. Multiply, divide and find powers of variables with exponents.

Example: Simplify $a^2b^6(a^3)$, $(n+2)\ (n-2)$, and $(n+2)^2$.

IM2.1.8 Add, subtract and multiply polynomials and divide polynomials by monomials.

Example: Subtract $(4x^2-7x+2)-(x^2+4x-5)$, multiply $(n+2)(4n-5)$ and divide $4x^3y^2+8xy^4-6x^2y^5$ by $2xy^2$.

IM2.1.9 Factor common terms from polynomials and factor quadratic expressions.

Example: Factor $4ax+3ay+4bx+3by$, $2x^2-7x+3$ and $9a^2-4$.

Standard 2

Geometry and Measurement

CORE STANDARDS
Angles and Lines

Understand the relationship between special angles created by parallel lines and transversals.

[Standard Indicator: IM2.2.2]

CORE STANDARDS
Congruence and Similarity

Develop simple geometric proofs involving congruent and similar polygons.

[Standard Indicator: IM2.2.6]

CORE STANDARDS
Geometric Proof and Reasoning

Understand the differences among supporting evidence, counterexamples and actual proofs. Be able to develop simple geometric proofs, providing reasons for each statement, involving the following topics:

- parallel lines and transversals;
- congruent and similar polygons, particularly triangles; and
- circles.

[Standard Indicators: IM2.2.6, IM2.2.23]

CORE STANDARDS
Triangles

Prove the Pythagorean Theorem and its converse and use them to solve problems. Define trigonometric functions in terms of angles of right triangles and use them to solve problems.

[Standard Indicators: IM2.2.13, IM2.2.16, IM2.2.18]

CORE STANDARDS
Circles

Define, deduce and use formulas for and prove theorems for:

- radius, diameter, arc, chord, secant and tangent;
- measures of arcs and related angles (central, inscribed, and intersections of secants and tangents); and
- circumference, arc length, and areas of circles and sectors.

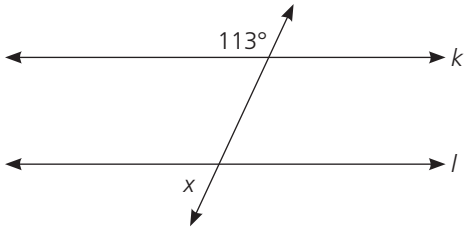
[Standard Indicators: IM2.2.20, IM2.2.21, IM2.2.22]

IM2.2.1 **Construct congruent segments and angles, angle bisectors, perpendicular bisectors, and parallel and perpendicular lines by using appropriate geometric construction tools. Explain and justify the process used.**

Example: Construct the perpendicular bisector of a given line segment, justifying each step of the process.

IM2.2.2 Recognize, use and justify the relationships between special pairs of angles formed by parallel lines and transversals.

Example: In the diagram, the lines k and l are parallel. What is the measure of angle x? Explain your answer.



IM2.2.3 Identify and apply properties of and theorems about parallel and perpendicular lines, write equations of parallel and perpendicular lines, and develop simple geometric proofs involving parallel and perpendicular lines.

Example: Find an equation of a line perpendicular to $y = 4x - 2$ that contains the point (4, 1).

IM2.2.4 Identify, justify and apply properties of planes.

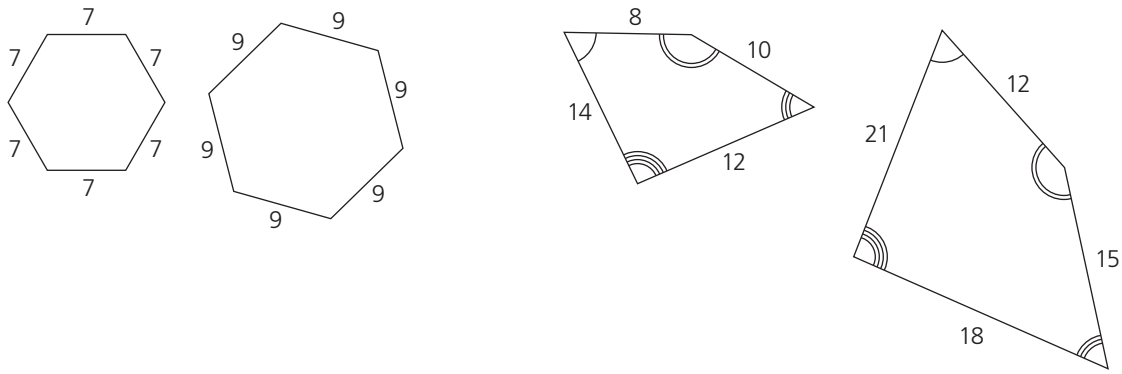
Example: Describe the intersection of plane R with parallel planes S and T.

IM2.2.5 Describe the intersection of two or more geometric figures in the plane.

Example: What is the maximum number of times two circles of the same size can intersect? Three circles? Six circles? Explain your reasoning.

IM2.2.6 Develop simple geometric proofs involving congruent and similar polygons and provide reasons for each statement.

Example: Prove that the following pairs of polygons are similar.



IM2.2.7 Describe, classify and recognize relationships among the quadrilaterals, such as squares, rectangles, rhombuses, parallelograms, trapezoids and kites.

Example: Use a drawing program to create a square, rectangle, rhombus, parallelogram, trapezoid and kite. Judge which of the quadrilaterals has perpendicular diagonals and draw those diagonals in the figures. Give a convincing argument that your judgment is correct.

IM2.2.8 Prove and apply theorems about parallelograms and trapezoids (including isosceles trapezoids) involving their angles, sides and diagonals. Prove that the given quadrilaterals are parallelograms, rhombuses, rectangles, squares or trapezoids (as appropriate).

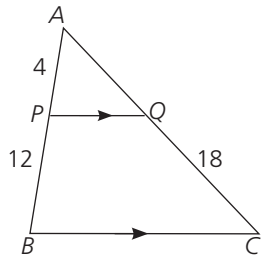
Example: Prove that both pairs of opposite sides of a parallelogram are congruent.

IM2.2.9 **Define, identify, construct and solve problems involving perpendicular bisectors, angle bisectors, medians and altitudes in triangles.**

Example: Draw several triangles. Construct their angle bisectors. What do you notice?

IM2.2.10 **Use theorems to show if two triangles are congruent (i.e., SSS, SAS, ASA) or similar (i.e., AA, SAS, SSS).**

Example: In the example below, prove that ΔABC and ΔAPQ are similar and use the similar triangles to compute the length of $\overline{AQ}$.

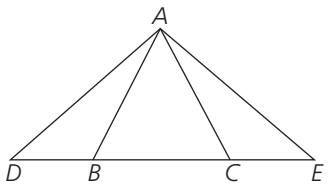


IM2.2.11 **Prove and apply the triangle inequality theorem.**

Example: Can 7, 15 and 5 be the sides of a triangle? Prove your answer is accurate.

IM2.2.12 **Prove and apply the isosceles triangle theorem and its converse.**

Example: Given isosceles ΔABC and $DB \cong EC$, prove $\Delta DBA \cong \Delta ECA$.



IM2.2.13 **Prove the Pythagorean Theorem and its converse and use them to solve problems, including problems involving the length of a segment in the coordinate plane.**

Example: Triangle DEF has vertices $D(2, 4)$, $E(0, 2)$, and $F(3, -1)$. Determine whether ΔDEF is a right triangle.

IM2.2.14 **Prove and apply the relationships that exist when an altitude is drawn to the hypotenuse of a right triangle.**

Example: In ΔABC with right angle at C, draw the altitude $\overline{CD}$ from C to $\overline{AB}$. Name all similar triangles in the diagram. Use these similar triangles to prove the Pythagorean Theorem.

IM2.2.15 **Use special right triangles (e.g., 30° - 60° and 45° - 45°) to solve problems.**

Example: An isosceles right triangle has one short side of 6 cm. Find the lengths of the other two sides.

IM2.2.16 **Define and use the trigonometric functions sine, cosine and tangent in terms of angles of right triangles.**

Example: In ΔABC, $\tan A = \frac{1}{5}$. Find $\sin A$ and $\cos A$.

IM2.2.17 **Deduce and apply the area formula $A = \frac{1}{2}ab \sin C$, where a and b are the lengths of two sides of a triangle and C is the measure of the included angle formed by the two sides.**

Example: Find the area of an equilateral triangle with sides five units long.

IM2

IM2.2.18 **Solve problems that can be modeled using right triangles, including problems that can be modeled using trigonometric functions. Interpret the solutions and determine whether the solutions are reasonable. Use technology as appropriate.**

Example: The force of gravity pulling an object down a hill is its weight multiplied by the sine of the angle of elevation of the hill. What is the force on a 3,000-pound car on a hill with a 1 in 5 grade? (A grade of 1 in 5 means that the hill rises one unit for every five horizontal units.)

IM2.2.19 **Construct the circle that passes through three given points not on a line. Construct tangents to circles. Circumscribe and inscribe circles. Justify the process used.**

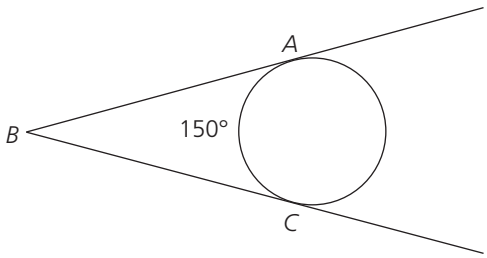
Example: Given a circle, find its center by drawing the perpendicular bisectors of two chords.

IM2.2.20 **Define, deduce and use formulas for, and prove theorems for radius, diameter, chord, secant and tangent.**

Example: What is the angle between a tangent to a circle and the radius at the point where the tangent meets the circle?

IM2.2.21 **Define, deduce and use formulas for, and prove theorems for measures of arcs and related angles (i.e., central, inscribed and intersections of secants and tangents).**

Example: Find the measure of $\angle ABC$ in the diagram below.



IM2.2.22 **Define, deduce and use formulas for, and prove theorems for measures of circumference, arc length, and areas of circles and sectors.**

Example: Use an appropriate theorem to find the sum of the interior angles of a convex n-gon.

IM2.2.23 **Develop simple geometric proofs involving circles and provide reasons for each statement.**

Example: Prove that an inscribed angle in a circle is half the measure of the central angle with the same arc.

IM2.2.24 **Identify, justify and apply properties of prisms, regular pyramids, cylinders, right circular cones and spheres.**

Example: Which of these properties of a cylinder is not true, and how do you know?

- The bases are congruent.
- The sections produced by the intersection of a cylinder and two parallel planes are congruent.
- The volume is the product of the area of the base and the altitude.
- The lateral area of a right circular cylinder is the product of the altitude and the base.

IM2.2.25 **Solve problems involving congruent and similar solids.**

Example: Explain how the surface area and volume of similar cylinders are related.

Standard 3

Data Analysis and Probability

CORE STANDARDS

Combinatorics and Probability

Use permutations, combinations and other counting methods to determine the number of ways that events can occur. Calculate the probability of compound events, and analyze probabilities to interpret odds and risks of events.

[Standard Indicators: IM2.3.6, IM2.3.7]

IM2.3.1 **For bivariate measurement data, create a scatter plot; describe its shape; and determine regression coefficients, regression equations and correlation coefficients using technological tools.**

Example: Measure the wrist and neck size of each person in your class and make a scatter plot. Interpret the correlation coefficient and the least squares regression line. Graph the residuals and evaluate the fit of the linear equation. Which line is a better fit? Explain your reasoning.

IM2.3.2 **Display and analyze bivariate data where at least one variable is categorical.**

Example: It is sometimes perceived that many retired people living in Florida live in mobile homes. Describe how you would gather, display and analyze the data to determine if more people 65 and older lived in mobile homes in Florida.

IM2.3.3 **Recognize how linear transformations of univariate data affect shape, center and spread.**

Example: Discuss whether you would use the mean or median to measure the center of each of the data below and why you made your particular choice.

- The yield of soybeans (bushels per acre) for a sample farm in Indiana.
- The prices of cars associated with each household in your neighborhood.

IM2.3.4 **Calculate and interpret the correlation coefficient. Use the correlation coefficient and residuals to evaluate a "best-fit" line.**

Example: Calculate and interpret the correlation coefficient for the linear regression model in the last example. Graph the residuals and evaluate the fit of the linear equation.

IM2.3.5 **Construct sample spaces and probability distributions in simple cases and use them to solve problems.**

Example: A couple plans to have children until they have a boy or until they have four children, whichever comes first. List the outcomes in the sample space for this experiment. What is the expected number of children for this couple?

IM2.3.6 **Determine the probability of simple events involving independent and dependent events and conditional probability. Analyze probabilities to interpret odds and risks of events.**

Example: When a die is rolled three times, what is the probability of obtaining a 6, followed by any even number, followed by a 4?

IM2.3.7 Use permutations, combinations and other counting methods to determine the number of ways that events can occur and to calculate probabilities, including the probability of compound events.

Example: There are five students who work in a bookshop. If the bookshop needs three people to operate, how many days straight could the bookstore operate without the same group of students working twice?

Standard 4

Discrete Mathematics

IM2.4.1 Use the properties of matrix addition, subtraction and scalar multiplication to solve problems.

Example: The table below shows the number of bound books produced during one shift at two publishing companies. Write a matrix to represent one day's total output at the two plants. Use your results to find the differences among production totals at the plants. Which plant produces more bound books with no graphics?

	Hardback With Graphic	Hardback No Graphic	Softback With Graphic	Softback No Graphic
Publisher A	500	800	950	1,900
Publisher B	600	700	1,000	1,600

IM2.4.2 Create matrices to organize and store data categorized by two variables and interpret the meaning of a particular entry in a matrix.

Example: Write a matrix C to represent the data in the table below. Find element $c_{24.}$ What does this element represent?

The Types of Books Chosen by Mr. Smith's Language Arts Classes				
	Fiction	Nonfiction	Science Fiction	Sports
Boys	15	20	18	22
Girls	18	17	10	15

IM2.4.3 Use the properties of matrix multiplication, including identity and inverse matrices, to solve problems.

Example: Explain how two matrices can be multiplied and what the dimensions of the product matrix will be.

IM2.4.4 Represent a system of equations in two or three variables as a matrix equation $A\vec{x} = \vec{b}$ and use technology to find $\vec{x} = A^{-1}\vec{b}$.

Example: Alana's Boutique is selling faux pearls for the following prices:

- 2 grey faux pearls and 3 black faux pearls cost $8.25.
- 3 grey faux pearls and 4 black faux pearls cost $11.25.

Let x = the cost of one grey pearl. Let y = the cost of one black pearl. Write the system as a matrix equation. Use technology to find the cost of one grey pearl and the cost of one black pearl.

IM2.4.5 Model and solve problems using matrices.

Example: To prepare for a dance, a school needs to rent 100 chairs, 4 large tables and 10 punch bowls. Rental prices were collected from two rental shops with the following matrix representing the two rental shops:

	R_1	R_2
Chairs	\$2	\$2.50
Tables	\$20	\$15
Bowls	\$6	\$4

Which rental shop, R_1 or R_2, has the lowest price for the group of items?

IM2.4.6 Use and interpret relational conjunctions (*and*, *or*, *not*), terms of causation (*if . . . then*) and equivalence *(if and only if*). Distinguish between the common uses of such terms in everyday language and their use in mathematics.

Example: Write "If today is Sunday, then we have school tomorrow" as a propositional statement by defining statements p and q and using symbols.

IM2.4.7 Use truth tables to determine the truth values of propositional statements.

Example: Find the truth table for $(p \rightarrow q) \rightarrow (q \rightarrow p)$.

IM2.4.8 Recognize syllogisms, tautologies, flawed reasoning and circular reasoning.

Example: Is the following reasoning valid? How do you know? "Today is Sunday, and we have school tomorrow. If today is not Sunday, then we have school tomorrow. Therefore, we do not have school tomorrow."

IM2.4.9 Construct and interpret directed and undirected graphs, decision trees, networks and flow charts.

Example: There are two islands in the River Seine in Paris. The city wants to construct four bridges that connect each island to each side of the riverbank and one bridge that connects the two islands directly. The city planners want to know if it is possible to start at one point, cross all five bridges, and end up at the same point without crossing a bridge twice. Use a graph to help solve this problem.

IM2.4.10 Use critical-path analysis to solve scheduling problems.

Example: Write a critical task list for redecorating your room. Some tasks depend on the completion of others and some may be carried out at any time. Use a graph to find the least amount of time needed to complete your project.

PROCESS STANDARDS

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The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
  - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
  - The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

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NOTES

Standard 1

Algebra and Functions

CORE STANDARDS
Quadratic Equations and Functions

Graphing Quadratic Functions
Graph quadratic functions. Determine how the graph of a parabola changes if *a*, *b*, and *c* are changed in the equation $y = a(x - b)^2 + c$. Find an equation for a parabola given sufficient information.

[Standard Indicators: IM3.1.4, IM3.1.6]

Solving Quadratic Equations
Solve quadratic equations in the complex number system. Solve problems that can be modeled using quadratic equations and functions.

[Standard Indicators: IM3.1.4, IM3.1.7]

CORE STANDARDS
Complex Numbers

Add, subtract, multiply and divide complex numbers.

[Standard Indicator: IM3.1.7]

CORE STANDARDS
Polynomial Equations and Functions

Writing Polynomial Equations
Perform arithmetic operations, including long division, on polynomials. Find a polynomial when given its roots and use the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph and factors of a polynomial expression to solve problems.

[Standard Indicators: IM3.1.10, IM3.1.14]

Solving Polynomial Equations
Solve polynomial equations by factoring. Solve problems that can be modeled using polynomial equations.

[Standard Indicators: IM3.1.11, IM3.1.13]

CORE STANDARDS
Rational Functions

Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator. Solve problems that can be modeled using equations involving rational functions.

[Standard Indicator: IM3.1.16]

CORE STANDARDS
Exponential and Logarithmic Equations

Use laws of exponents to derive laws of logarithms. Use laws of logarithms to solve problems. Solve exponential and logarithmic equations. Solve problems that can be modeled using equations involving exponents and logarithms.

[Standard Indicators: IM3.1.22, IM3.1.23, IM3.1.24]

CORE STANDARDS
Sequences and Series

Find specific terms of arithmetic and geometric sequences. Find partial sums of arithmetic and geometric series. Solve problems that can be modeled using arithmetic and geometric series.

[Standard Indicators: IM3.1.25, IM3.1.26, IM3.1.27]

IM3.1.1 **Find the zeros, domain and range of a function.**

Example: $f(x) = x^3 - 3x^2 - x + 3$. What is the domain and range of this function?

IM3.1.2 **Define, add, subtract, multiply and divide complex numbers. Represent complex numbers and the addition, subtraction and absolute value of complex numbers in the complex plane.**

Example: Let $z = 7 - 4i$ and $w = 10 + 6i$. Graph z, w and $z + w$. Prove that the number $0, z, w$ and $z + w$ are the vertices of a parallelogram on the complex plane.

IM3.1.3 **Solve quadratic equations in the complex number system.**

Example: Solve $x^2 - 2x + 5 = 0$ over the complex numbers.

IM3.1.4 **Analyze, describe and sketch graphs of quadratic functions and include the lines of symmetry.**

Example: Find the zeros for $y = x^2 - 4$. If $y = x^2 - 4$ has a maximum or minimum value, give the ordered pair corresponding to the maximum or minimum point.

IM3.1.5 Determine how the graph of a parabola changes if a, b and c changes in the equation $y = a(x-b)^2 + c$. Find an equation for a parabola when given sufficient information.

Example: Write the equation of the parabola with vertex (3, 6), y-intercept 2 in vertex form.

IM3.1.6 Solve problems that can be modeled using quadratic equations and functions, interpret the solutions, and determine whether the solutions are reasonable.

Examples:

- Write the equation of the parabola with vertex (3, 6) and y-intercept 2 in vertex form.
- Describe similarities and differences in the graphs of $y = 2x^2$ and $y = 2(x-1)^2 + 3$ without first graphing the equations.

IM3.1.7 Analyze, describe and sketch graphs of polynomial functions by examining intercepts, zeros, domain and range, and end behavior.

Example: Determine by inspection the end behavior of the graph of the function $f(x) = -2x^3 + x^2 + 4x - 5$.

IM3.1.8 Use the binomial theorem to expand binomial expressions raised to positive integer powers.

Example: Expand $(x+2)^4$.

IM3.1.9 Perform arithmetic operations, including long division and division with remainders, on polynomials by others of equal or lower degree.

Example: Divide $2x^3 - 3x^2 + x - 6$ by $x^2 + 2$.

IM3.1.10 Factor polynomials completely and solve polynomial equations by factoring.

Example: Solve $x^3 + 27 = 0$ by factoring.

IM3.1.11 Use graphing technology to find approximate solutions for polynomial equations.

Example: Approximate the solution(s) of $x^4 - 3x^3 + 2x - 7 = 0$ to the nearest tenth.

IM3.1.12 Solve problems that can be represented or modeled using polynomial equations, interpret the solutions and determine whether the solutions are reasonable.

Example: You want to make an open-top box with a volume of 500 cubic inches from a piece of cardboard that is 25 inches by 15 inches by cutting squares from the corners and folding up the sides. Then use your results to give a formula for the volume of the box.

IM3.1.13 Find a polynomial function of lowest degree with real coefficients when given its roots. Solve problems by using the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph and factors of a polynomial expression.

Example: Write an equation that has solutions $x = 2$, $x = 5i$, and $x = -5i$.

IM3.1.14 Analyze, describe and sketch graphs of rational functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: Find the equations of the horizontal and vertical asymptotes of the function $f(x) = \dfrac{x+1}{x+5}$.

IM3.1.15 Add, subtract, multiply, divide, reduce and evaluate rational expressions with polynomial denominators. Simplify rational expressions, including expressions with negative exponents in the denominator.

Example: Simplify $\dfrac{x^2-4}{x^5} \div \dfrac{x^3-8}{x^8}$.

IM3.1.16 Understand the properties of rational exponents and use the properties to simplify, multiply, divide and find powers of expressions containing negative and fractional exponents. Relate expressions containing rational exponents to the corresponding radical expressions.

Example: Write the expression $\left(x^{\frac{1}{2}}\, y^{\frac{-2}{3}}\right)^6$ in simplest form. Assume all variables are positive.

IM3.1.17 Analyze, describe and sketch graphs of square root and cube root functions by examining intercepts, zeros, domain and range, and end behavior.

Example: Graph the function $y = \sqrt{x} + 7$ and find the domain and range.

IM3.1.18 Solve equations that contain radical expressions and identify extraneous roots when they occur.

Example: Solve the equation $x = \sqrt{x} + 2$.

IM3.1.19 Solve problems that can be modeled using equations involving rational and radical functions, including problems of direct and inverse variation. Interpret the solutions and determine whether the solutions are reasonable.

Example: Two students working independently can complete a particular job in 20 minutes and 30 minutes, respectively. How long will it take to complete the job if they work together at the same rate as when doing the job alone?

IM3.1.20 Analyze, describe and sketch graphs of exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.

Example: Draw the graphs of the functions $y = 2^x$ and $y = 2^{-x}$.

IM3.1.21 Know that the inverse of an exponential function is a logarithm. Use laws of exponents to derive laws of logarithms. Use the inverse relationship between exponential functions and logarithms and the laws of logarithms to solve problems.

Example: If you know that $\log(2) = a$ and $\log(3) = b$, find $\log(36)$ in terms of a and b.

IM3.1.22 Solve exponential and logarithmic equations.

Example: Solve the equation $\log_2 x = 5$.

IM3.1.23 Solve problems that can be modeled using exponential and logarithmic equations, interpret the solutions, and determine whether the solutions are reasonable. Use technology as appropriate.

Example: The population of a certain country can be modeled by the equation $P(t) = 50e^{0.02t}$, where P is the population in millions and t is the number of years after 1900. Find when the population is 100 million, 200 million and 400 million. What do you notice about these time periods?

IM3.1.24 Write the recursive formula for arithmetic and geometric sequences and find specified terms of arithmetic and geometric sequences.

Example: Find the tenth term of the arithmetic sequence 3, 7, 11, 15… .

IM3.1.25 Write the formula for the general term for arithmetic and geometric sequences and make connections to linear and exponential functions.

Example: Write the formula for the general term of the geometric sequence 2, 6, 18, 54, 162… .

IM3.1.26 Find partial sums of arithmetic and geometric series.

Example: In the last example (IM3.1.25), find the sum of the first 10 terms.

IM3.1.27 Solve problems involving applications that can be modeled using sequences and finite arithmetic and geometric series. Interpret the solutions and determine whether the solutions are reasonable using spreadsheets as appropriate.

Example: A restaurant has square tables that seat four people. When two tables are placed together, six people can be seated. If 20 square tables are placed together to form one long table, how many people can be seated?

Standard 2

Geometry and Measurement

CORE STANDARD
Coordinate Geometry

Find slopes, lengths and midpoints of line segments using coordinate geometry. Use these measures to show whether shapes are similar or congruent and whether line segments are parallel or perpendicular.

[Standard Indicators: IM3.2.1, IM3.2.5]

CORE STANDARD
Triangles

Develop simple geometric proofs involving triangles.

[Standard Indicator: IM3.2.4]

CORE STANDARD
Circles

Find the equation of a circle in the coordinate plane. Determine how the graph of a circle changes if a, b and r are changed in the equation $(x - a)^2 + (y - b)^2 = r^2$.

[Standard Indicator: IM3.2.5]

CORE STANDARD
Solids

Find and use measures of sides, volumes of solids and surface areas of solids. Relate these measures to each other using formulas.

[Standard Indicator: IM3.2.6]

IM3.2.1 **Represent geometric objects and figures algebraically using coordinates, use algebra to solve geometric problems, and develop simple coordinate proofs involving geometric objects in the coordinate plane.**

Example: Draw a triangle with vertices at (1, 3), (2, 5) and (6, 1). Draw another triangle with vertices (-3, -1), (-2, 1) and (2, -3). Are these triangles the same shape and size? Justify your answer.

IM3.2.2 Construct triangles congruent to given triangles. Explain and justify the process used.

Example: Construct a triangle given the lengths of two sides and the measure of the angle between the two sides.

IM3.2.3 Develop simple geometric proofs involving triangles and provide reasons for each statement of the proof.

Example: Prove the following:

- The sum of the angles in a triangle is 180°.
- The line joining the midpoint of two sides of a triangle is parallel to, and half the length of, the third side.
- The perpendicular bisectors of the sides of a triangle meet at a point that is the center of the circle and that contains the vertices of the triangle.

IM3.2.4 Find the equation of a circle in the coordinate plane in terms of its center and radius and determine how the graph of a circle changes if a, b and r are changed in the equation $(x-a)^2+(y-b)^2=r^2$.

Example: Find the equation of the circle with radius 10 and center (6, -3).

IM3.2.5 Find and use measures of sides, volumes and surface areas of prisms, regular pyramids, cylinders, right circular cones and spheres. Relate these measures to each other using formulas.

Example: A marble is dropped into a glass that is roughly a right cylinder with a 6 cm diameter. The water level rises 1 mm. What is the volume of the marble?

IM3.2.6 Visualize solids and surfaces in three-dimensional space when given two-dimensional representations, and create two-dimensional representations for the surfaces of three-dimensional objects.

Example: Make a net for a tetrahedron out of poster board and fold it to make the tetrahedron.

IM3.2.7 Describe the structure of and relationships within an axiomatic system (e.g., undefined terms, definitions, axioms and postulates, methods of reasoning, and theorems).

Example: Do you prove axioms from theorems or theorems from axioms?

IM3.2.8 Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true. Illustrate its counterparts in other geometries.

Example: Describe and illustrate at least one non-Euclidean geometry postulate.

Standard 3

Data Analysis and Statistics

IM3.3.1 Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions.

Example: About 30 percent of the students at a school are on the honor roll. If you took a random sample of 30 students, what range of students would likely be on the honor roll?

IM3.3.2 Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis and the validity of conclusions. Interpret confidence levels and "margin of error."

Example: In a random poll of 1,025 women, it was found that 47 percent of the women polled said they do not get enough time for themselves. The poll announced a margin of error of ±3 percent points for 95 percent confidence in its conclusion. Explain to someone who knows no statistics why it cannot be said that 47 percent of all adult women do not get enough time for themselves. Then explain what "95 percent confidence" means.

IM3.3.3 Compare the differences among surveys, experiments and observational studies and recognize which types of inferences can legitimately be drawn from each.

Example: Gather data to answer the question: Which area of the country has the highest high school dropout rate? Display your dropout data in various forms.

IM3.3.4 Compute basic statistics (mean, median, weighted mean, range, variance, standard deviation) and understand the distinction between a statistic and a parameter.

Example: Use spreadsheet formulas to compute measures that summarize your dropout data by state.

IM3.3.5 Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term *variable*.

Example: Compare the data displayed in various forms in the first example. What do you notice about the impact the type of display has on the analysis of the data?

IM3.3.6 Use simulations to construct empirical probability distributions.

Example: Describe how you could use two coins to set up a simulation of a random phenomenon that has a 25 percent chance of a desired outcome.

IM3.3.7 Know and apply the characteristics of the normal distribution.

- Identify settings in which the normal distribution may be useful.
- Determine whether a set of data appears to be uniform, skewed or normally distributed.
- Use the empirical rule to find probabilities that an event will occur in a specific interval that can be described in terms of one, two or three standard deviations from the mean.

Example: Math SAT scores are normally distributed with mean 500 and standard deviation 100. What is the probability that a randomly selected student's SAT score is greater than 600?

IM3.3.8 Compute and interpret the expected value of random variables in simple cases.

Example: When you flip a coin five times, the number of heads is 0, 1, 2, 3, 4 or 5. Find the probability of each number of heads and draw a histogram of the results.

IM3.3.9 Compute the probability of compound events.

Example: An experiment consists of rolling a die three times and noting the number that lands on top at each throw. Find the probability of a 6, followed by any even number, followed by a 4.

IM3.3.10 Model and solve problems, including probability problems, using counting techniques.

Example: You know that your locker combination contains the numbers 2, 4, 6 and 8, but you have forgotten the order in which they occur. What is the probability that your locker opens with the first combination you try?

Standard 4

Discrete Mathematics

IM3.4.1 Know and use the concepts of sets, elements and subsets.

Example: In the set of integers { … , -3, -2, -1, 0, 1, 2, 3… }, write the subset of even integers.

IM3.4.2 Perform operations on sets (union, intersection, complement, cross product).

Example: In the set of integers from 1 to 10, a subset is the prime numbers. Name the elements in this subset's complement.

IM3.4.3 Identify and give examples of undefined terms, axioms and theorems.

Example: Do you prove axioms from theorems or theorems from axioms?

IM3.4.4 Describe logical statements using the terms *assumption*, *hypothesis*, *conclusion*, *converse*, *inverse* and *contrapositive*. Find the converse, inverse and contrapositive of statements.

Example: Find the converse of "If today is Sunday, then we have school tomorrow." Is the converse logically equivalent to the original statement? Explain.

IM3.4.5 Explain and illustrate the role of definitions, conjectures, theorems, proofs and counterexamples in mathematical reasoning. Construct logical arguments, assess the validity of logical arguments and give counterexamples to disprove statements.

Example: Find an example to show that triangles with two sides and one angle equal are not necessarily congruent.

IM3.4.6 Model and solve problems involving patterns using recursion and iteration, growth and decay, and compound interest.

Example: How many handshakes would occur in this room if everyone shook hands with everyone else? Create a spreadsheet that will find the number of handshakes starting with one person and increasing the number to 50.

IM3.4.7 Use mathematical induction to prove simple propositions.

Example: Use mathematical induction to prove the sum of the first n even positive integers is $n(n + 1)$.

IM3.4.8 Use graph-coloring techniques to solve problems.

Example: Color a map of the Midwestern states of the United States so that no adjacent states are the same color. What is the minimum number of colors needed?

IM3.4.9 Use bin-packing techniques to solve problems of optimizing resource usage.

Example: Six large crates of electronic equipment are to be shipped to a warehouse. The crates weigh 2,800, 6,000, 5,400, 1,600, 6,800 and 5,000 pounds. Each delivery truck has a capacity of 10,000 pounds. What is the minimum number of trucks needed to send all the crates?

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Standard 1

Relations and Functions

CORE STANDARD
Graphing Functions

Use paper and pencil methods and graphing technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric and inverse trigonometric functions. Identify domain, range, intercepts, zeros, asymptotes and points of discontinuity of functions. Use graphs to solve problems.

[Standard Indicators: PC.1.1, PC.1.2, PC.1.3, PC.3.1, PC.3.2, PC.3.3, PC.4.8, PC.4.9, PC.4.10]

CORE STANDARD
Logarithmic and Exponential Functions

Define and find inverse functions. Verify whether two given functions are inverses of each other. Solve problems involving logarithmic and exponential functions by using the laws of logarithms and understand why those properties are true.

[Standard Indicators: PC.1.7, PC.3.2]

PC.1.1 **Use paper and pencil methods and technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric, inverse trigonometric and piecewise-defined functions. Use these graphs to solve problems, and translate among verbal, tabular, graphical and symbolic representations of functions by using technology as appropriate.**

Example: Draw the graphs of the functions $y = x^5 - 2x^3 - 5x^2$, $y = \dfrac{2x-1}{3x+2}$, $y = \sqrt{(x+2)(x-5)}$ and $f(x) = \sin^{-1}x$.

PC.1.2 **Identify domain, range, intercepts, zeros, asymptotes and points of discontinuity of functions represented symbolically or graphically, using technology as appropriate.**

Example: Let $R(x) = \dfrac{1}{\sqrt{x-2}}$. Find the domain of $R(x)$ (i.e., the values of x for which $R(x)$ is defined).

Also, find the range and asymptotes of $R(x)$.

PC.1.3 **Solve word problems that can be modeled using functions and equations.**

Example: You are on the committee for planning the prom and need to decide what to charge for tickets. Last year you charged \$5.00 and 400 people bought tickets. Earlier experiences suggest that for every 20-cent decrease in price you will sell 20 extra tickets. Use a spreadsheet and write a function to show how the amount of money in ticket sales depends on the number of 20-cent decreases in price. Construct a graph that shows the price and gross receipts. What ticket price maximizes revenue?

PC.1.4 Recognize and describe continuity, end behavior, asymptotes, symmetry and limits and connect these concepts to graphs of functions.

Example: Determine the numbers a and b so that the following function is continuous.

$$f(x) = \begin{cases} x^2 \text{ if } x \le 1 \\ ax + b \text{ if } 1 < x < 2 \\ 5 - x \text{ if } x \ge 2 \end{cases}$$

PC.1.5 Find, interpret and graph the sum, difference, product and quotient (when it exists) of two functions and indicate the relevant domain and range of the resulting function.

Example: Find $(f+g)(x)$ if $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{x}{x-1}$. State the domain of $(f+g)(x)$.

PC.1.6 Find the composition of two functions and determine the domain and the range of the composite function. Conversely, when given a function, find two other functions for which the composition is the given one.

Example: If $h(x) = (2x+3)^4$, find functions f and g so that $f \circ g = h$.

PC.1.7 Define and find inverse functions, their domains and their ranges. Verify symbolically and graphically whether two given functions are inverses of each other.

Example: Find the inverse function of $h(x) = (x-2)^3$.

PC.1.8 Apply transformations to functions and interpret the results of these transformations verbally, graphically and numerically.

Example: Explain how you can obtain the graph of $g(x) = -\left|2(x+3)^2 - 2\right|$ from the graph of $f(x) = x^2$.

Standard 2

Conics

CORE STANDARD

Conic Sections

Derive equations for conic sections. Graph conic sections by hand by completing the square and find foci, centers, asymptotes, eccentricity, axes and vertices as appropriate.

[Standard Indicators: PC.2.1, PC.2.2]

PC.2.1 Derive equations for conic sections and use the equations that have been found.

Example: Derive an equation for the ellipse with foci at (-1, 0) and (1, 0) that contains the point (0, 2).

PC.2.2 Graph conic sections with axes of symmetry parallel to the coordinate axes by hand, by completing the square, and find the foci, center, asymptotes, eccentricity, axes and vertices (as appropriate).

Example: Graph $\dfrac{(x-2)^2}{4} - \dfrac{(y+3)^2}{9} = 1$. Find its foci, centers, asymptotes, eccentricity, axes and vertices (as appropriate).

Standard 3

Logarithmic and Exponential Functions

CORE STANDARD
Graphing Functions

Use paper and pencil methods and graphing technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric and inverse trigonometric functions. Identify domain, range, intercepts, zeros, asymptotes and points of discontinuity of functions. Use graphs to solve problems.

[Standard Indicators: PC.1.1, PC.1.2, PC.1.3, PC.3.1, PC.3.2, PC.3.3, PC.4.8, PC.4.9, PC.4.10]

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Logarithmic and Exponential Functions

Define and find inverse functions. Verify whether two given functions are inverses of each other. Solve problems involving logarithmic and exponential functions by using the laws of logarithms and understand why those properties are true.

[Standard Indicators: PC.1.7, PC.3.2]

PC.3.1 **Compare and contrast symbolically and graphically $y = e^x$ with other exponential functions.**

Example: Graph $y = e^x$, $y = 3^x$ and $y = 2^{-x}$. Show how to rewrite 3^x and 2^{-x} as e^{kx} for certain values of k.

PC.3.2 **Define the logarithmic function $g(x) = \log_a x$ as the inverse of the exponential function $f(x) = a^x$. Apply the inverse relationship between exponential and logarithmic functions and apply the laws of logarithms to solve problems.**

Example: Simplify the expression $e^{\ln 8}$.

PC.3.3 **Analyze, describe and sketch graphs of logarithmic and exponential functions by examining intercepts, zeros, domain and range, and asymptotic and end behavior.**

Example: For the function $l(x) = \log_{10}(x - 4)$, find its domain, range, x-intercept and asymptote, and sketch the graph.

PC.3.4 **Solve problems that can be modeled using logarithmic and exponential functions. Interpret the solutions and determine whether the solutions are reasonable.**

Example: The amount A of a radioactive element (in gm) after t years is given by the formula: $A(t) = 100e^{-0.02t}$. Find t when the amount is 50 gm, 25 gm and 12.5 gm. What do you notice about these time periods?

Standard 4

Trigonometry

CORE STANDARD
Unit Circle

Define sine and cosine using the unit circle. Convert between degree and radian measures. Use the values of the sine, cosine and tangent functions at 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ and $\frac{\pi}{2}$ radians and their multiples.

[Standard Indicators: PC.4.4, PC.4.5, PC.4.6]

CORE STANDARD
Trigonometric Functions

Define and analyze trigonometric functions, including inverse functions. Solve problems involving trigonometric functions and prove trigonometric identities.

[Standard Indicators: PC.4.8, PC.4.9, PC.4.10, PC.4.11]

CORE STANDARD
Graphing Functions

Use paper and pencil methods and graphing technology to graph polynomial, absolute value, rational, algebraic, exponential, logarithmic, trigonometric and inverse trigonometric functions. Identify domain, range, intercepts, zeros, asymptotes and points of discontinuity of functions. Use graphs to solve problems.

[Standard Indicators: PC.1.1, PC.1.2, PC.1.3, PC.3.1, PC.3.2, PC.3.3, PC.4.8, PC.4.9, PC.4.10]

PC.4.1 **Define and use the trigonometric ratios cotangent, secant and cosecant in terms of angles of right triangles.**

Example: Use the relationships among the lengths of the sides of a 30° - 60° right triangle to find the exact value of the secant of 30°.

PC.4.2 **Model and solve problems involving triangles using trigonometric ratios.**

Example: Find the area of ΔABC if a, which is the side opposite angle A, measures 5 units; b, which is the side opposite angle B, measures 8 units; and angle C, which is the angle opposite c, measures 30°.

PC.4.3 **Develop and use the laws of sines and cosines to solve problems.**

Example: You want to determine the location of a water tower by taking measurements from two positions three miles apart. From the first position, the angle between the water tower and the second position is 78°. From the second position, the angle between the water tower and the first position is 53°. How far is the water tower from each position?

PC.4.4 Define sine and cosine using the unit circle.

Example: Find the acute angle A for which $\sin 150° = \sin A$.

PC.4.5 Develop and use radian measures of angles, measure angles in degrees and radians, and convert between degree and radian measures.

Examples:

- Convert 90°, 45° and 30° to radians.
- Find the length of an arc subtended by an angle of $\frac{5\pi}{6}$ radians on a circle of radius 5 cm.

PC.4.6 Deduce geometrically and use the value of the sine, cosine and tangent functions at 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$ and $\frac{\pi}{2}$ radians and their multiples.

Example: Find the values of $\cos \frac{\pi}{2}$, $\tan \frac{3\pi}{4}$, $\csc \frac{2\pi}{3}$, $\sin^{-1} \frac{\sqrt{3}}{2}$ and $\sin 3\pi$.

PC.4.7 Make connections among right triangle ratios, trigonometric functions and the coordinate function on the unit circle.

Example: Angle A is a 60° angle of a right triangle with a hypotenuse of length 14 and a shortest side of length 7. Find the exact sine, cosine, and tangent of angle A. Find the real numbers x, $0 < x < 2\pi$, with exactly the same sine, cosine and tangent values.

PC.4.8 Analyze and graph trigonometric functions, including the translation of these trigonometric functions. Describe their characteristics (i.e., spread, amplitude, zeros, symmetry, phase, shift, vertical shift, frequency).

Example: Draw the graph of $y = 5 + \sin\left(x - \frac{\pi}{3}\right)$.

PC.4.9 Define, analyze and graph inverse trigonometric functions and find the values of inverse trigonometric functions.

Example: Graph $f(x) = \sin^{-1} x$.

PC.4.10 Solve problems that can be modeled using trigonometric functions, interpret the solutions and determine whether the solutions are reasonable.

Example: In Indiana, the length of a day in hours varies through the year, usually with the longest day of about 14 hours on June 21 and the shortest day of about 10 hours on December 21. Model this situation with a sine function, by giving both the graph of this function and its formula. Find another day that is as long as July 4 by using your model.

PC.4.11 Derive the fundamental Pythagorean trigonometric identities; sum and difference identities; half-angle and double-angle identities; and the secant, cosecant and cotangent functions. Use these identities to verify other identities and simplify trigonometric expressions.

Example: Find the acute angle between the lines given by $y = 2x$ and $y = 3x$.

PC.4.12 Solve trigonometric equations and interpret solutions graphically.

Example: Solve $3 \sin 2x = 1$ for x between 0 and 2π.

Standard 5

Polar Coordinates and Complex Numbers

CORE STANDARD

Polar Coordinates and Complex Numbers

Define and use polar coordinates and complex numbers. Graph equations in the polar coordinate plane. Use their relation to trigonometric functions to solve problems.

[Standard Indicators: PC.5.1, PC.5.2, PC.5.3, PC.5.4]

PC.5.1 **Define and use polar coordinates and relate polar coordinates to Cartesian coordinates.**

Example: Convert the polar coordinate $(2, \frac{\pi}{3})$ to Cartesian coordinates.

PC.5.2 **Represent equations given in Cartesian coordinates in terms of polar coordinates.**

Example: Represent the equation $x^2 + y^2 = 4$ in terms of polar coordinates.

PC.5.3 **Graph equations in the polar coordinate plane.**

Example: Graph $y = 1 - \cos \theta$.

PC.5.4 **Define complex numbers, convert complex numbers to polar form and multiply complex numbers in polar form.**

Example: Write $3 + 3i$ and $2 - 4i$ in trigonometric form and then multiply the results.

PC.5.5 **Prove and use De Moivre's Theorem.**

Example: Simplify $(1 - i)^{23}$.

Standard 6

Sequences and Series

CORE STANDARD

Sequences and Series

Define arithmetic and geometric sequences and series. Prove and use the sum formulas for arithmetic series and for finite and infinite geometric series. Describe the concept of the limit of a sequence and a limit of a function. Decide whether simple sequences converge or diverge and recognize an infinite series as the limit of a sequence of partial sums. Use series to solve problems. Derive the binomial theorem by combinatorics.

[Standard Indicators: PC.6.1, PC.6.2, PC.6.3]

PC.6.1 **Define arithmetic and geometric sequences and series.**

Example: Write an example and explain the differences among each of the following sequences and series: arithmetic sequence, geometric sequence, arithmetic series and geometric series.

PC.6.2 **Derive and use formulas for finding the general term for arithmetic and geometric sequences.**

Example: Write the general term formula for the arithmetic sequence with initial term two and common difference three.

PC.6.3 **Develop, prove and use sum formulas for arithmetic series and for finite and infinite geometric series.**

Example: Find the sum of the infinite geometric series $8 + 4 + 2 + \ldots$.

PC.6.4 **Generate a sequence by using recursion.**

Example: Write the first five terms of the Fibonacci sequence with $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$. Observe a pattern of even and odd terms for this sequence. Prove that your observation is correct.

PC.6.5 **Describe the concept of the limit of a sequence and a limit of a function. Decide whether simple sequences converge or diverge. Recognize an infinite series as the limit of a sequence of partial sums.**

Example: Find the limit as $n \to \infty$ of the sequence $\frac{2n-1}{3n+2}$ and the limit as $x \to 5$ of the function $\frac{x^2 - 5^2}{x - 5}$.

PC.6.6 **Model and solve word problems involving applications of sequences and series, interpret the solutions and determine whether the solutions are reasonable.**

Example: You put one grain of rice on the square of a chess board the first day, two on a second square the next day and continue to double the number of grains of rice you place each day through the 64^{th} day. How many grains of rice need to be used on the 64^{th} square on the 64^{th} day? How many grains of rice were used all together?

PC.6.7 **Derive the binomial theorem by combinatorics.**

Example: Give an algebraic and combinatorial proof of the binomial theorem if a and b are real numbers and n is a positive integer.

Standard 7

Vectors and Parametric Equations

PC.7.1 **Define vectors as objects having magnitude and direction. Represent vectors geometrically.**

Example: Graph the vector that goes from point $p = (3, 2)$ to the point $q = (5, 6)$.

PC.7.2 **Use parametric equations to represent situations involving motion in the plane.**

Example: Car A is traveling east at 40 mph, and Car B is traveling north at 30 mph. Both are heading toward the same intersection. Car A is five miles from the intersection when car B is four miles from the intersection. Write the parametric equations that describe the position of each car as a function of time.

PC.7.3 **Convert between a pair of parametric equations and an equation in x and y.**

Example: Given the parametric equations $x = 3t^2$ and $y = t + 1$, find an equation relating x and y.

PC.7.4 **Analyze planar curves, including those given in parametric form.**

Example: Describe the curve that is defined by the parametric equations $x = 5 \cos t$ and $y = 5 \sin t$ for $0 \leq t \leq \pi$.

PC.7.5 Model and solve problems using parametric equations.

Example: For the problem in Standard 7.2, write a formula for the distance between the cars as a function of time and then find when the cars are the closest.

Standard 8

Data Analysis

PC.8.1 Find linear models by using median fit and least squares regression methods. Decide which among several linear models gives a better fit. Interpret the slope in terms of the original context.

Example: Measure the wrist and neck size of each person in your class and make a scatter plot. Find the median fit line and the least squares regression line. Which line is a better fit? Explain your reasoning.

PC.8.2 Calculate and interpret the correlation coefficient. Use the correlation coefficient and residuals to evaluate a "best-fit" line.

Example: Calculate and interpret the correlation coefficient for the linear regression model in the last example. Graph the residuals and evaluate the fit of the linear equation.

PROCESS STANDARDS

Indiana's Academic Standards for Mathematics describe the key content of each grade level and course, and students must develop conceptual understanding of this content. The American Diploma Project noted that, "beyond acquiring procedural mathematical skills with their clear methods and boundaries, students need to master the more subjective skills of reading, interpreting, representing and 'mathematicizing' a problem" (p. 55).

The National Council of Teachers of Mathematics has described five Process Standards that "highlight ways of acquiring and using content knowledge" (p. 29). The following Process Standards must be addressed throughout the learning and teaching of Indiana's Academic Standards for Mathematics in all grade levels in mathematics.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof

- Recognize reasoning and proof as fundamental aspects of mathematics.
- Make and investigate mathematical conjectures.
- Develop and evaluate mathematical arguments and proofs.
- Select and use various types of reasoning and methods of proof.

Communication

- Organize and consolidate mathematical thinking through communication.
- Communicate mathematical thinking coherently and clearly to peers, teachers and others.
- Analyze and evaluate the mathematical thinking and strategies of others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- Recognize and apply mathematics in contexts outside of mathematics.

Representation

- Create and use representations to organize, record and communicate mathematical ideas.
- Select, apply and translate among mathematical representations to solve problems.
- Use representations to model and interpret physical, social and mathematical phenomena.

In addition, estimation, mental computation and technology are areas that need to be addressed at all grade levels in mathematics.

Estimation and Mental Computation

- Know and apply appropriate methods for estimating the results of computations.
- Use estimation to decide whether answers are reasonable.
- Decide when estimation is an appropriate strategy for solving a problem.
- Determine appropriate accuracy and precision of measurement in problem situations.
- Use properties of numbers and operations to perform mental computation.
- Recognize when the numbers involved in a computation allow for a mental computation strategy.

Technology

- Technology should be used as a tool in mathematics education to support and extend the mathematics curriculum.
- Technology can contribute to concept development, simulation, representation, communication and problem solving.
- The challenge is to ensure that technology supports, but is not a substitute for, the development of skills with basic operations, quantitative reasoning, and problem-solving skills.
  - Graphing calculators should be used to enhance middle school and high school students' understanding and skills.
  - The focus must be on learning mathematics and using technology as a tool rather than as an end unto itself.

References

American Diploma Project (2004). *Ready or Not: Creating a High School Diploma that Counts*. Washington, D.C.: Achieve, Inc.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*. Reston VA: author.

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